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A TWO-PHASE MODEL TO DESCRIBE THE BEHAVIOUR OF SATURATED GRANULAR-COHESIVE SOIL MIXTURES

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ABSTRACT

Alpine moraines and glacial tills are typical examples of heterogeneous soils or *melanges*, containing particles of widely disparate size. This paper presents a constitutive model, based on a two-component mixture to describe their mechanical behaviour in saturated conditions. Results of triaxial and one-dimensional compression tests, performed on artificial mixtures of gravel and kaolin in different proportions, are then used to validate the material model. It is shown that, if the parameters for the granular fraction are properly selected, the model may provide suitable predictions of the mechanical response of these heterogeneous soils.

Key-words: heterogeneous soils, melanges, constitutive modelling

INTRODUCTION

Glacial tills and Alpine moraines are examples of soils containing particles with dimensions varying among several orders of magnitude, ranging from cobble to clay size. These materials are classified as heterogeneous soils or *melanges* and may be basically schematised as a two-component mixture, in which the granular fraction (GF) interacts with a fine fraction (FF) matrix, the latter permeated by liquid and/or gassy phases (Mitchell, 1976). Their overall properties depend on the grading and shape of the coarse particles, on the fine mineralogy, on the water content and especially on the GF and FF percentages. Figure 1 shows the bi-phase representation of the granular-clay mixture.

Experimental studies (Fukue et al. 1986, Omine et al. 1994, Thevanayam and Mohan 2000, Wood and Kumar 2000) have shown that if the FF exceeds 70-80% the *melange* response is influenced by the FF behaviour, due to the absence of any interaction between the particles of a larger size. When the GF increases, its influence becomes significant and, beyond a certain threshold level, the GF completely determines the overall response. Many difficulties arise in modelling the *melange* behaviour for similar GF and FF contents: simplifying assumptions have been therefore introduced in previous studies in order to account for the stress/strain relationship between fractions, but without always providing remarkable results.

This study presents the results of a preliminary laboratory investigation consisting of triaxial and 1D-compression tests on artificial mixtures composed of two basic artificial components, namely a calcareous gravel and a kaolin, mixed with each other in various proportions. A material model belonging to the Cam-Clay type family, modified to take into account the influence of the melange composition, was then used to model the experimental data.

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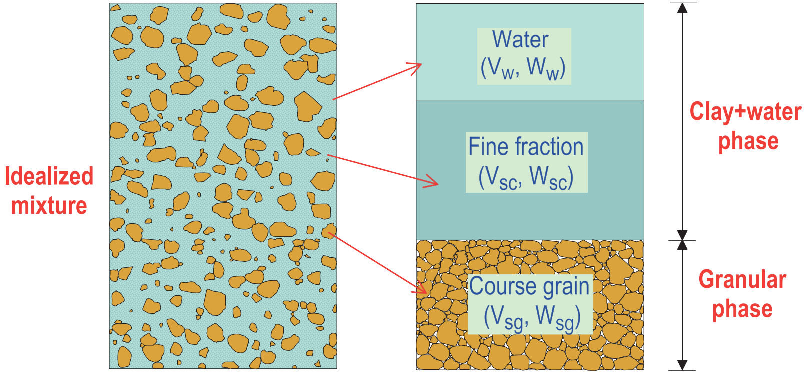

Fig 1: Two-phase representation of granular soil-clay mixture

VOLUMETRIC VARIABLES

In saturated mixtures of two basic components, i.e. the GF and the FF, the fine content C may be defined as the percentage of FF mass in relation to the total solid mass. Since the specific gravity of the constituents is usually very similar C can be equally expressed as a volume ratio.

Given that V_w, V_C, and V_G represent the volume of water, FF and GF respectively, and e represents the mixture void ratio, two other void ratios, namely the granular and matrix void ratios e_G and e_C, may be defined as:

$$e_G = \frac{V_W + V_C}{V_G} = \frac{100e + C}{100 - C} \tag{1a}$$

$$e_C = \frac{V_W}{V_C} = \frac{100e}{C} \tag{1b}$$

At a high C, GF grains are not in contact and stresses divide themselves between the two fractions according to the relative component stiffness. As C decreases, the possibility that two or more grains come into contact rapidly increases: when this occurs, the grains form a stiffer skeleton and the FF trapped inside is unable to deform or to support stress increments, thus becomes a part of the skeleton. When $C=C_r$, all the grains are in contact and the soil behaves as a granular material.

As suggested by Omine et al. (1994), it is convenient to distinguish two portions of FF and GF: one constituting the skeleton (V'_w, V'_C, and V'_G) and one related to the matrix (V''_w, V''_C, and V''_G). Note that V''_G is the volume of grains completely immersed in the matrix. Consistent with this assumption, the volume fractions of the matrix f_C and skeleton R are defined as:

$$f_C = \frac{V''_W + V''_C}{V} \tag{2a}$$

$$R = \frac{V'_W + V'_C + V'_G}{V} \tag{2b}$$

It easy to demonstrate that:

$$f_C = 1 - \left\{ 1 + \left(1 + e_{C0}\right) \left[\frac{C}{100 - C} - \frac{C_r}{100 - C_r} \right] \right\}^{-1} \tag{3}$$

where e_{co} is the matrix void ratio at C=100%.
On the basis of a probability analysis of the assemblages of the grains in contact, Omine (2002) suggested that R could be related to f_C through this simple relationship:

$$R = (1 - f_C)^2 \tag{4}$$

Note that at C=100% the volume fractions become f_C=1 and R=0, but when C=C_r the presence of the matrix vanishes and it is f_C=0 and R=1.

THE TWO-PHASE MODEL

In order to evaluate the splitting of stresses into the two soil components and the properties of a mixture, Omine et *al*. (1994) introduced a stress distribution parameter b, defined by the following equations:

$$\mathrm{d}q_{\mathrm{C}} = b\mathrm{d}q_{\mathrm{G}} \tag{5a}$$

$$\mathrm{d}p'_{\mathrm{C}} = b\mathrm{d}p'_{\mathrm{G}} \tag{5b}$$

where dq and dp' are increments of the deviatoric stress q and mean effective stress p' with "G" and "C" subscripts referring to the skeleton and matrix respectively.
The increments dq and dp' and the shear and volumetric strains $\mathrm{d}\varepsilon_q$ and $\mathrm{d}\varepsilon_p$ of the melange are:

$$\begin{cases} \mathrm{d}q = R\mathrm{d}q_{\mathrm{G}} + (1-R)\mathrm{d}q_{\mathrm{C}} \\ \mathrm{d}p' = R\mathrm{d}p'_{\mathrm{G}} + (1-R)\mathrm{d}p'_{\mathrm{C}} \end{cases} \tag{6a,b}$$

$$\begin{cases} \mathrm{d}\varepsilon_{\mathrm{q}} = R\mathrm{d}\varepsilon_{\mathrm{q,G}} + f_{\mathrm{C}}\mathrm{d}\varepsilon_{\mathrm{q,C}} \\ \mathrm{d}\varepsilon_{\mathrm{p}} = R\mathrm{d}\varepsilon_{\mathrm{p,G}} + f_{\mathrm{C}}\mathrm{d}\varepsilon_{\mathrm{p,C}} \end{cases} \tag{7a,b}$$

In equations (7a-b) no distinction is made whether strains are of an elastic or plastic nature, because different conditions may occur: the components can deform both in an elastic or plastic state, or the skeleton deformations could be elastic whereas the matrix ones could be plastic or *viceversa*.
The parameter b is the ratio between stress increments. If both fractions are isotropic and elastic, b is a constant and quantities q and p' may substitute increments dq and dp' in (5a,b). In all other situations b depends on the loading conditions and on the singular specific volumes.
To determine b it is assumed, in this context, that increments of strain occur with an equal distribution of internal work between the two fractions. Moreover, an elasto-plastic modified Cam-Clay type model (Roscoe and Burland 1968) is assumed as a constitutive model for both fractions.

Elastic conditions

In elastic isotropic compression, the specific volume v=1+e (e = void ratio) is linearly related to the natural logarithm of the mean stress p'. If κ is the e-ln$p$' line slope, the elastic volumetric strain and the corresponding skeleton and matrix internal works are:

$$\mathrm{d}\varepsilon_{\mathrm{p}}^{\mathrm{e}} = \frac{\kappa}{v}\frac{\mathrm{d}p'}{p'} \tag{8a}$$

$$\mathrm{d}W_{\mathrm{G}} = p'_{\mathrm{G}}\,\mathrm{d}\varepsilon_{\mathrm{p,G}}^{\mathrm{e}} = p'_{\mathrm{G}}\frac{\kappa_{\mathrm{G}}}{v_{\mathrm{G}}}\frac{\mathrm{d}p'_{\mathrm{G}}}{p'_{\mathrm{G}}} = b\frac{\kappa_{\mathrm{G}}}{v_{\mathrm{G}}}\mathrm{d}p'_{\mathrm{C}} \tag{8b}$$

$$\mathrm{d}W_{\mathrm{C}} = p'_{\mathrm{C}}\,\mathrm{d}\varepsilon_{\mathrm{p,C}}^{\mathrm{e}} = \frac{\kappa_{\mathrm{C}}}{v_{\mathrm{C}}}\mathrm{d}p'_{\mathrm{C}} \tag{8c}$$

Imposing the equivalence of expressions (8a) and (b) the parameter b is:

$$b = \frac{\kappa_{\mathrm{C}}}{\kappa_{\mathrm{G}}}\frac{v_{\mathrm{G}}}{v_{\mathrm{C}}} \tag{9}$$

If both components deform elastically, it is assumed that $p'_{\mathrm{G}}=b\cdot p'_{\mathrm{C}}$. Using equation (7b), the overall volumetric strain can be expressed as:

$$\mathrm{d}\varepsilon_{\mathrm{p}}^{\mathrm{e}} = R\mathrm{d}\varepsilon_{\mathrm{p,G}}^{\mathrm{e}} + f_{\mathrm{C}}\mathrm{d}\varepsilon_{\mathrm{p,C}}^{\mathrm{e}} = (R\frac{\kappa_{\mathrm{G}}}{v_{\mathrm{G}}}\frac{\mathrm{d}p'_{\mathrm{G}}}{p'_{\mathrm{G}}} + f_{\mathrm{C}}\frac{\kappa_{\mathrm{C}}}{v_{\mathrm{C}}}\frac{\mathrm{d}p'_{\mathrm{C}}}{p'_{\mathrm{C}}}) = (R\frac{\kappa_{\mathrm{G}}}{v_{\mathrm{G}}} + f_{\mathrm{C}}\frac{\kappa_{\mathrm{C}}}{v_{\mathrm{C}}})\frac{\mathrm{d}p'_{\mathrm{C}}}{p'_{\mathrm{C}}} \tag{10}$$

Finally, by combining the relationships (10), (8a) and (6b), one obtains:

$$\frac{\kappa}{v} = R\frac{\kappa_{\mathrm{G}}}{v_{\mathrm{G}}} + f_{\mathrm{C}}\frac{\kappa_{\mathrm{C}}}{v_{\mathrm{C}}} \tag{11}$$

Under pure shear within the elastic range the increments of the shear strain and internal work for the two fractions are:

$$\mathrm{d}\varepsilon_{\mathrm{q}}^{\mathrm{e}} = \frac{\mathrm{d}q}{3G} \tag{12a}$$

$$\mathrm{d}W_{\mathrm{G}} = q_{\mathrm{G}}\mathrm{d}\varepsilon_{\mathrm{q,G}}^{\mathrm{e}} = q_{\mathrm{G}}\frac{\mathrm{d}q_{\mathrm{G}}}{3G_{\mathrm{G}}} = b^2\frac{q_{\mathrm{C}}\mathrm{d}q_{\mathrm{C}}}{3G_{\mathrm{G}}} \tag{12b}$$

$$\mathrm{d}W_{\mathrm{C}} = q_{\mathrm{C}}\mathrm{d}\varepsilon_{\mathrm{q,C}}^{\mathrm{e}} = \frac{q_{\mathrm{C}}\mathrm{d}q_{\mathrm{C}}}{3G_{\mathrm{C}}} \tag{12c}$$

In this case the parameter b and the overall shear modulus become:

$$b = \left(G_{\mathrm{G}}/G_{\mathrm{C}}\right)^{0.5} \tag{13a}$$

$$G = \left(Rb+1-R\right)\Bigg/\left(\frac{Rb}{3G_{\mathrm{G}}} + \frac{f_{\mathrm{C}}}{3G_{\mathrm{C}}}\right) \tag{13b}$$

Plastic conditions

In the modified Cam-Clay model the yield surface and the plastic strains for a general stress increment exceeding the elastic region are:

$$p' = \frac{M^2}{M^2 - \eta^2} p'_o \tag{14a}$$

$$\mathrm{d}\varepsilon_p^p = \frac{\lambda - \kappa}{v} \frac{\mathrm{d}p'_o}{p'_o} \tag{14b}$$

$$\mathrm{d}\varepsilon_q^p = \frac{2\eta}{M^2 - \eta^2} \mathrm{d}\varepsilon_p^p \tag{14c}$$

in which p'_o represents the current size of the elliptical yield surface. By assuming that $\varpi = (\lambda - \kappa)/v$ and $T = M^2/(M^2 - \eta^2)$ the expressions of the plastic work and the parameter b become:

$$\mathrm{d}W^p = q\mathrm{d}\varepsilon_q^p + p'\mathrm{d}\varepsilon_p^p = \varpi T \mathrm{d}p'_o \tag{15a}$$

$$b = \frac{\varpi_C}{\varpi_G} \frac{T_C}{T_G} = \frac{\varpi_C}{\varpi_G} \tag{15b}$$

Note that in equation (15b) it is assumed that $T = T_C = T_G$. This means that both singular components and mixture are in an equivalent state in relation to the critical condition – i.e. $\eta/M = \eta_G/M_G = \eta_C/M_C$. Combining (14b-c), (7b) and (10) the quantities ϖ and λ/v for an hydrostatic load increment result in:

$$\varpi = R\varpi_G + f_C\varpi_C \tag{16a}$$

$$\frac{\lambda}{v} = R\frac{\lambda_G}{v_G} + f_C\frac{\lambda_C}{v_C}. \tag{16b}$$

The shear strain of mixture can be written as:

$$\mathrm{d}\varepsilon_q^p = \frac{2\eta}{M^2 - \eta^2} \varpi \frac{\mathrm{d}p'_o}{p'_o} = R\mathrm{d}\varepsilon_{q,G}^p + f_C\mathrm{d}\varepsilon_{q,C}^p = \left(R\frac{2\eta_G\varpi_G}{{M_G}^2 - {\eta_G}^2} + f_C\frac{2\eta_C\varpi_C}{{M_C}^2 - {\eta_C}^2} \right) \frac{\mathrm{d}p'_o}{p'_o} \tag{17}$$

Multiplying both members by T, the above relationship supplies:

$$\frac{\varpi}{\eta} = R\frac{\varpi_G}{\eta_G} + f_C\frac{\varpi_C}{\eta_C}. \tag{18}$$

Finally, combining together (18), (16a) and (15b), the slope M of the critical state line can be expressed as:

$$M = \frac{(Rb + f_C)}{\left(\frac{Rb}{M_G} + \frac{f_C}{M_C} \right)} \tag{19}$$

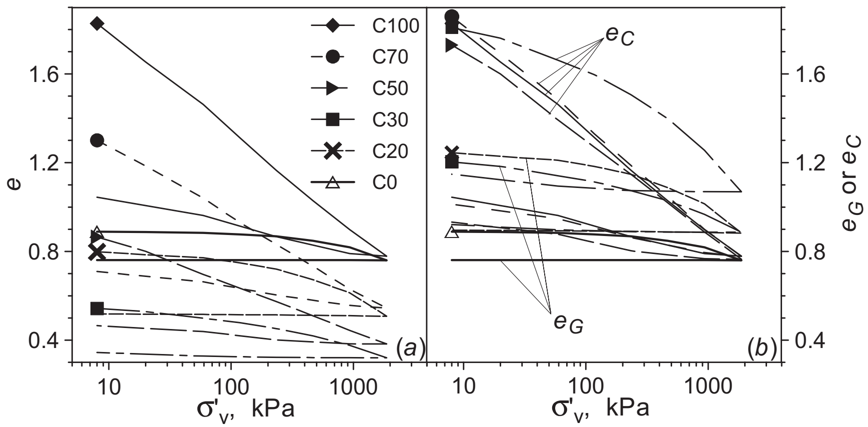

Fig 2a,b: Void ratios of mixtures in the oedometer tests.

EXPERIMENTAL PROGRAMME

The two selected mixture components consist of a sub-angular calcareous gravel (D_{50}=4.87mm, U=D_{60}/D_{10}=1.48 and $G_{s,G}$=2.70) and a kaolin (D_{50}=1,3µm, CF=58, LL=59, LP=39 and $G_{s,C}$=2.63), which were mixed at a water content equal to 1,5÷1,8LL. The tests were performed on mixtures with different kaolin content, namely C = 0, 20, 30, 50, 70 and 100%. In the paper, the letter C and relative kaolin content is used to label the corresponding test. Pure gravel specimens were tested in the loosest state, obtained by pluvial deposition.

In one-dimensional compression tests the material was spooned into the oedometer cell with care to avoid any air being trapped in the samples with a high fine content. They were then consolidated up to a vertical stress of 1.8 MPa. The initial sample height was around 5 cm and the diameter 8.0 cm. The results of one-dimensional compression tests are shown in Figure 2a in terms of e-log(σ'_v), whereas in Figure 2b they are reported in terms of the e_G or e_C.

Triaxial specimens were prepared by consolidating at a vertical stress of 100 kPa the mixture in a mould, with an internal diameter of 7.0 cm and a height of about 14 cm. In the triaxial cell the specimens were then anisotropically consolidated with η=0.62 at a constant strain rate of 0.018 %/min up to a maximum vertical stress of 150 kPa. Undrained shear was then applied at an axial strain rate of $7 \cdot 10^{-4}$ %/min. The results are shown in Figures 3a,b, in terms of q vs. axial strain ε_a, excess pore pressure u vs. ε_a; effective stress-path in the plane q-p' is also sketched in the same figure.

MODEL VERIFICATION

The model described above is completely defined by four independent parameters, namely λ, κ, G and M which are derived from the same parameters of the two basic components and by the matrix volume ratio f_c, which, in turn, is a function of e_{C0} and C_r. All the model parameters can be determined from standard laboratory tests.

The 1D test results show that e gradually decreases as C decreases up to C of about 30%. If C decreases below this value, e increases again but not regularly. In figure 2b the e_C-log(σ'_v) curves of tests C100, C70 and C50 are superimposed, thus confirming that the overall volume

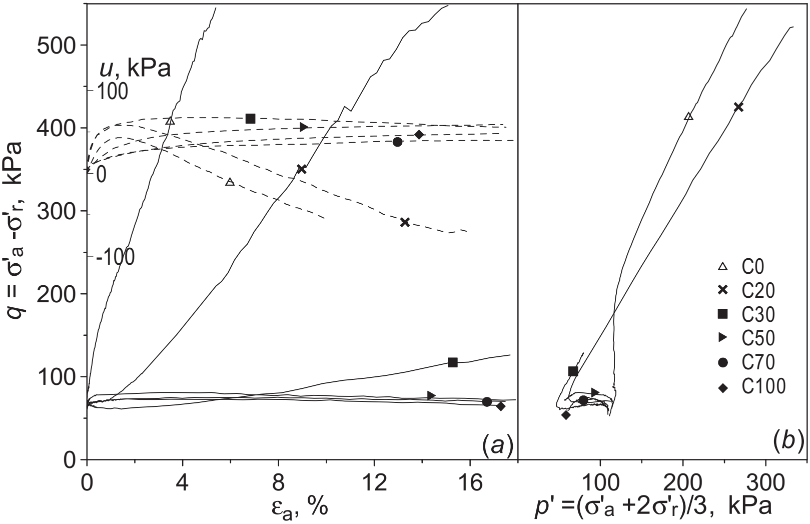

Fig. 3a,b: Undrained shear in triaxial compression tests for different gravel-kaolin mixtures.

variation at a high C is due only to the matrix compression. On the contrary, the e_G-log(σ'_v) curves of tests $C30$ and $C20$ are very close each other, indicating that in these tests the compressibility is almost completely controlled by the GF behaviour. Note that, for C=30%, both e and e_G depend on the initial granular soil density and, therefore, on the procedure of sample preparation, as it usually occurs in granular material: among the specimens with low C, the sample $C20$ shows the highest e and e_G.

In Figure 4 the void ratio e at σ'_v=100 kPa is plotted against C. For C>30% the experimental results fit well equation (1b) with e_C=1.35 (line a). On the contrary, for C<30% the equation (1a) – line b - calibrated on the test $C0$ with e_G=0.88 (which is the value of e correspondent on

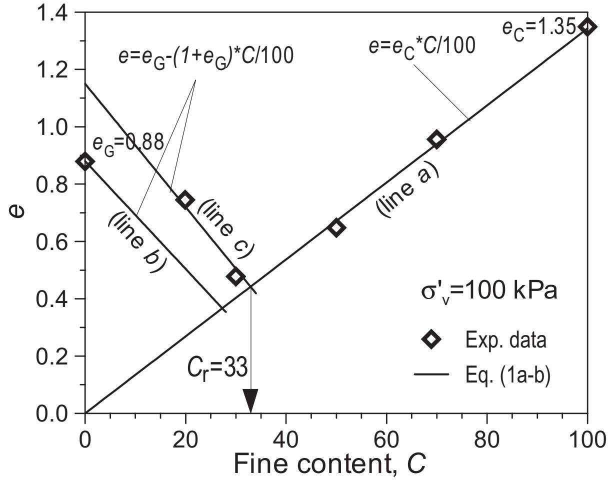

Fig. 4: Void ratio of the mixture as a function of fine content.

Table 1 – Experimental model parameters.

Mixture	M (ϕ_{crit})	e at 100 kPa	κ/v	λ/v	ϖ	G_o (MPa)
C100	0.97 (24.9°)	1.35	0.025	0.086	0.060	37
C20	1.59 (38.9°)	0.75	0.0012	0.035	0.033	73
C0	1.94 (47.1°)	0.88	0.0002	0.016	0.015	105

pure gravel) fails when predicting the void ratio for *C*20 and *C*30. These values of e are fitted by the same equation (1a), but assuming e_G=1.15 as reference value (line *c*).

This means that, in a mixture such as that tested in this investigation, the coarse grains begin to come in contact, only when e_G is greater than the typical values of e provided by test *C*0, the latter prepared in the loosest density obtainable with the usual laboratory procedure. Consequently, it was preferred to assume in the model the properties derived from test *C*20, including the void e_G =1.15, that represent the reference values of GF. In figure 4, from the intersection of lines (*a*) and (*c*), it results $C_r \cong 33\%$.

Table 1 summarises the mechanical properties for the matrix and granular phases. In column two is also reported (between parenthesis) the critical friction angle, which is the strength parameter customarily used in geotechnical engineering practice. The friction angle can be easily determined from the critical strength parameter M by ϕ'_{crit}=arcsin[3M/(6+M)].

Figures 5a and 5b show the experimental values of λ/v and κ/v against C, where λ and κ are evaluated in the range σ'_v=100-1000 kPa and v is the specific volume at σ'_v=100 kPa. Due to the non-linear response of specimens with C=30%, the slope was evaluated, in this case, by using the difference between the current void ratio at σ'_v=100 and at σ'_v=1000 kPa.

It is interesting to note in Figures 5a and 5b that the constitutive relationships (16b) and (11) fit well the experimental results, if the results of the test *C*0 are excluded from the analysis, as discussed above.

Figures 6a and 6b report the experimental values of the small-strain shear modulus G_0 and the

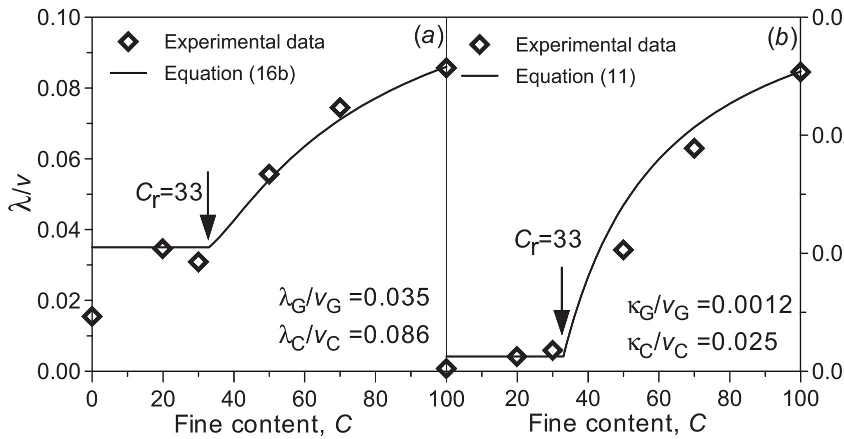

Fig. 5a,b: Parameters λ and κ as function of fine content *C*.

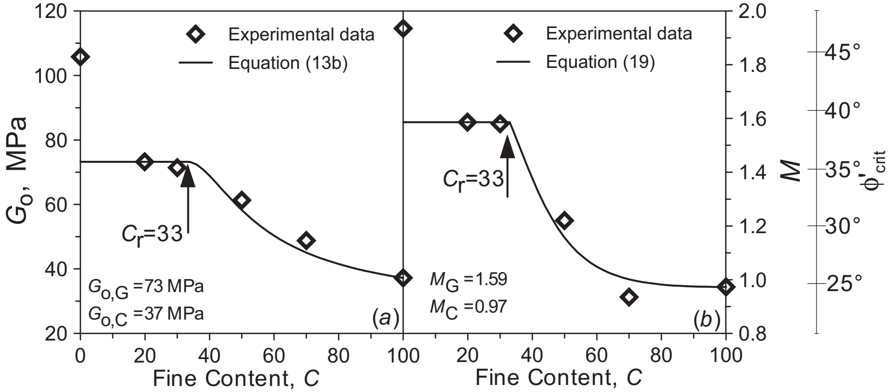

Fig. 6a,b: Parameters G_0 and M as function of fine content C.

strength parameter M (on the right there is also the axis for the critical friction angle ϕ'_{crit}) determined in the triaxial tests. Also for these data, it is interesting to notice that the model is able to describe the variation of G_0 and M with C, with the exception of the values provided by the test C0, the latter values being very high compared to those given by the test C20.

The above differences cannot be completely explained considering the high difference in the relative density of the coarse-grain skeleton in test C0 (e_G=0.64) and in test C20 (e_G=0.86). This, in fact, may justify the higher value of G_0 (affected by relative density) but not that of M, because the critical state condition is unique for a given soil, independent from density value.

Some problems were also encountered in performing the triaxial test for the pure gravel specimen, namely for test C0. To avoid the penetration of the rubber membrane into the large voids among gravel grains (and therefore the membrane failure), three membranes were installed all around the specimen: the standard correction for membrane penetration was, of course, taken into account in the interpretation of test results, but other unknown effects may probably have occurred. Moreover, due to the small range of the load cell installed in the triaxial device, the test C0 was terminated before reaching the critical state condition, as it is clearly evident from the graphs of Figure 3a and 3b. Consequently, the value of M reported in Figure 6b may be greater than the slope of the actual critical state line.

CONCLUSIONS

The two-component mixture model, opportunely modified to take into account the behaviour change when coarse grains come into contact, is a good base for building up an analytical model for predicting heterogeneous soil properties.

The preliminary formulation presented here is based on the elasto-plastic Cam-Clay modified constitutive model coupled with a criterion of equal distribution of the internal work between the two fractions. The latter was introduced to describe the stress/strain relationship between fractions.

From the experimental investigation carried out so far and the related constitutive modelling it appears that if the parameters for the granular fraction are adequately selected, the model can suitably predict the overall properties of the mixture.

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