Prediction of Variation in Reservoir Sedimentation by Gravimetry Technique

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The study is aimed at accurate and fast estimation of variation in reservoir sediment due to torrential rains by gravimetry technique. The related algorithms and software are well analyzed and investigated. Before practical gravity surveys, we firstly propose several simulated conditions of Tseng-Wen reservoir. They are conditions with high, medium, and low water levels and deposit levels. Gravity variations derived by terrain corrections (TC) method are approximately 0.01 - 0.20 mgal, depending on different reservoir conditions. Practical gravity surveys will be completed during typhoon season this year, in order to evaluate our simulated results described in this paper. Absolute gravimeters are more recommended for the following gravity surveys than relative gravimeters. The purpose of the research is to develop a fast and accurate method to estimate the sediment variations in reservoirs, and subsequently bring contributions to soil and water conservation.

Key words: gravimetry, reservoir sediment, soil and water conservation.

1. INTRODUCTION

Taiwan experiences couples of typhoon hazards which usually bring torrential rains, particularly during summer seasons. These disasters occasionally cause big landslides and subsequently a large amount of sediments fall into reservoirs. According to the report from Water Resources Agency of Taiwan, there were totally 94 reservoirs in Taiwan in 2008. The pondages of all reservoirs reached 2,082,359,000 m^3. However, Typhoon Morakot dramatically reduced the capacities of the reservoir storages in 2009. For example, 93,000,000 tons of deposits went into Tseng-Wen reservoir, and it resulted in 30% loss of reservoir storage capacity.

Although sediments in reservoirs can be predicted by use of remote sensing techniques (e.g., LiDAR or depthometer), most of them are time-consuming methods. Thus, gravimetry technique could be taken into consideration for fast, accurately, and efficiently estimating sediment variation of a reservoir before and after landslides. The related theories and softwares for this study are preliminarily completed, and some experiments by using simulated data have been carried out. They are both detailedly and entirely described in this paper.

2. METHOLOGY

A method for computing terrain corrections (TC) is used in this study for predicting variation of reservoir sedimentation. TC is a procedure removing the gravity effect of topographic variation at P (**Fig. 1**). In **Fig. 1**, the mass of part A, which attracts upward, is removed, causing gravity at P to increase. The mass deficiency of part B is made up, causing gravity at P to increase again [*Heiskanen and Moritz*, 1967]. The total gravity effects to P from masses A and B are called the effects of TC. In a planar approximation, the potential V due to the topographic mass above and below a point at P (x_p, y_p, s) (see **Fig. 1**) is given by *Heiskanen and Moritz* [1967] and *Torge* [1989].

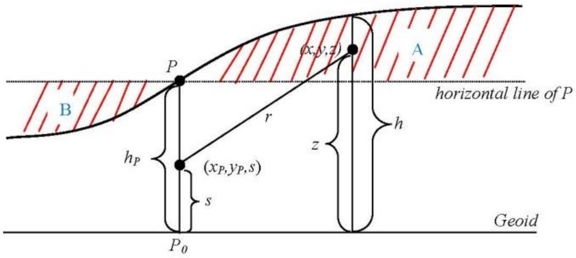

Fig. 1 Geometry of terrain correction

$$V = G\rho\int_x\int_y\int_{z=h_p}^{h}\frac{dxdydz}{\sqrt{(x-x_p)^2+(y-y_p)^2+(z-s)^2}} \quad (1)$$

where G is the gravitational constant ($G = 6.67221937\times10^{11} m^3 kg^{-1} s^{-2}$) and ρ is the density (e.g., water: 1.0 g/cm^3, and rock: 2.67 g/cm^3). The vertical attraction at P due to this potential is TC and can be expressed as *Hwang et al*. [2003].

$$tc = (\frac{\partial V}{\partial s})\Big|_{s=h_p} = G\rho\int_x\int_y\int_{z=h_p}^{h}\frac{(z-h_p)dxdydz}{[(x-x_p)^2+(y-y_p)^2+(z-h_p)^2]^{3/2}}$$

$$= G\rho\int_x\int_y[\frac{1}{\sqrt{(x-x_p)^2+(y-y_p)^2}}$$

$$-\frac{1}{\sqrt{(x-x_p)^2+(y-y_p)^2+(h-h_p)^2}}]dxdy$$

$$= G\rho\int_x\int_y f(x,y)dxdy \quad (2)$$

We used Gaussian quadrature method [*Evans*, 1993] to execute Eq. (2). Gaussian quadrature is a rigorous point wise method and Eq. (2) can be rewritten as

$$G\rho\int_{X_1}^{X_2}\int_{Y_1}^{Y_2} f(x,y)dxdy \approx G\rho\sum_{j=1}^{M} w_j^y c(y_j) \quad (3)$$

and

$$c(y) = \int_{X_1}^{X_2} f(x,y)dx \approx \sum_{i=1}^{N} w_i^x f(x_i, y) \quad (4)$$

Where X_1, X_2, Y_1 and Y_2 are given area boundary. w_i^x and w_j^y are weighting coefficients, x_i and y_j are nodal coordinates, M and N are the numbers of weighting coefficients and nodes along the x and y axes over the domains $[X_1, X_2]$ and $[Y_1, Y_2]$. In this study, X_1, X_2, Y_1, Y_2 are west, east, south, and north of a given DEM. M and N are the numbers of the given DEM grids along x and y.

In **Fig. 1**, if point P is near a reservoir, the TC values could be obviously changed due to the variations of water or deposits before and after tremendous rains or landslides. Therefore, we can estimate the amount of reservoir storage or sediment variations by analyzing the differences between two TC values both computed at point P. For example, tc_1 and tc_2 represent the TC values observed at P before and after a typhoon disaster. The change of TC can be easily obtained by

$$\Delta tc = tc_1 - tc_2 \quad (5)$$

Then the amount of water or deposit in the reservoir after the typhoon disaster can be precisely estimated.

TC has been applied in various regions in the past decades, such as geoid computation [*Nahavandchi*, 2000], interpretation of crustal structure [*Camacho et al.*, 2001] and orthometric correction [*Hwang and Hsiao*, 2003]. In this study, we propose a program (tcq_rsv.f) in FORTRAN 90 based on Gaussian quadrature to compute the TC effects by simulating several reservoir conditions.

3. A CASE STUDY IN TSENG-WEN RESERVOIR

3.1 Test Area and Data

The study area in this paper is Tseng-Wen reservoir, which is the rectangle displayed in **Fig. 2(a)**. Tseng-Wen reservoir is located in mountain areas of Chiayi County, south of Taiwan. Its surrounding areas are about 250 - 1000 m height. **Fig. 2(b)** shows a zoomed-in view picture of the whole reservoir. Tseng-Wen reservoir forms the largest reservoir in Taiwan. The total capacity is around 708,000,000 m^3. The surface and catchment areas are 17 and 481 km^2, respectively. In **Fig. 2(b)**, the rectangle denotes the dam area and its zoom-in view is shown in **Fig. 2(c)**. In this study, we select three simulated gravity stations, which are 100, 200,

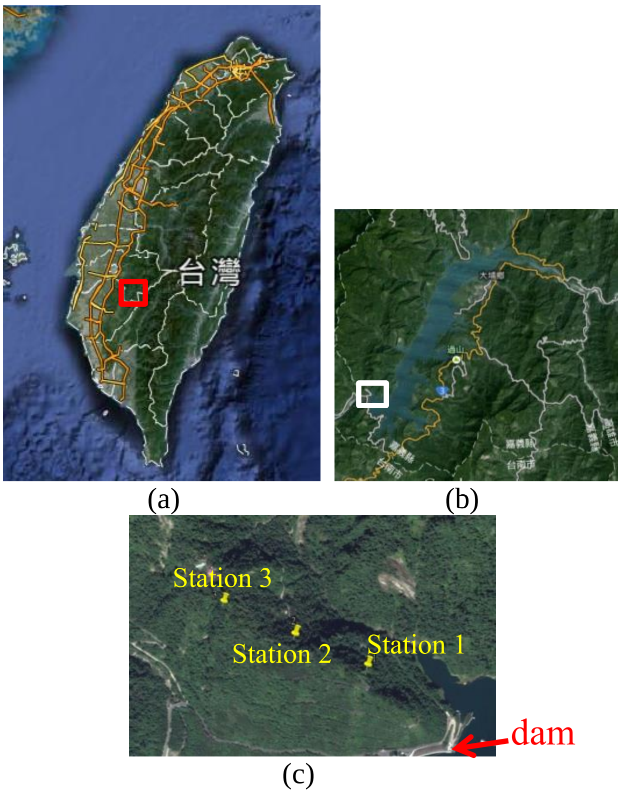

Fig. 2 (a) The location (The red rectangle) of Tseng-Wen reservoir. (b) A zoomed-in view of the red rectangle. (c) A zoomed-in view of the white rectangle. The three yellow dots in (c) are the selected simulated gravity stations from near to far. All figures are extracted from google earth.

and 300 m away from the dam. Their locations are also represented in **Fig. 2(c)**. We assume that gravity values are observed in these points (as point P in Eqs. (1) and (2)). Furthermore, a $3'' \times 3''$ DEM generated by photogrammetry is used to help TC computations.

3.2 Design of experiment

Different conditions of water and deposit levels for Tseng-Wen reservoir are considered for the gravity investigation. We design three different levels of water surfaces, which are 275 m (high), 265 m (medium), and 255 m (low), and also three different levels of deposit surfaces, which are 251 m (high), 248 m (medium), and 245 m (low). The cases divided by different conditions of water and deposit levels are shown in **Table 1**. For water level, cases 1/4/7, cases 2/5/8, and cases 3/6/9 belong to low, medium, and high, respectively; for deposit, low, medium, and high levels are classified to cases 1/2/3, cases 4/5/6, and cases 7/8/9, respectively. **Figs. 3 and 4** show the $3'' \times 3''$ DEMs around Tseng-Wen reservoir with high, medium, and low water levels, and high, medium, and low deposit levels, respectively. The elevations of north-south and east-west going profiles are shown in **Fig. 5**. The three black dots in **Fig. 3** and white ones in **Fig. 4** both located in the bottom left corners represent the positions of three gravity stations. Compared to **Fig. 3(a) and (b)**, the pondage of the high water level has 170,000,000 m^3 more than that of medium one. The pondage difference between **Fig. 3(b) and (c)** is also 170,000,000 m^3. Moreover, the differences of sediment volumes between **Fig. 4(a) and (b)**, and **Fig. 4(b) and (c)**, are approximately both 51,000,000 m^3 (around 68,000 tons). In this study, we assume that the densities of water and sediments are 1.0 and 1.35 g/cm^3, respectively.

The TC value of each case will be computed. Moreover, the relation between the simulated water or sediment variation, and the TC difference between every 2 cases will be also analyzed. The related results are all summarized in section 4.

Table 1 Cases with different conditions of water and deposit levels for Tseng-Wen reservoir.

Case	Water level	Deposit level
case1	low	low
case2	medium	low
case3	high	low
case4	low	medium
case5	medium	medium
case6	high	medium
case7	low	high
case8	medium	high
case9	high	high

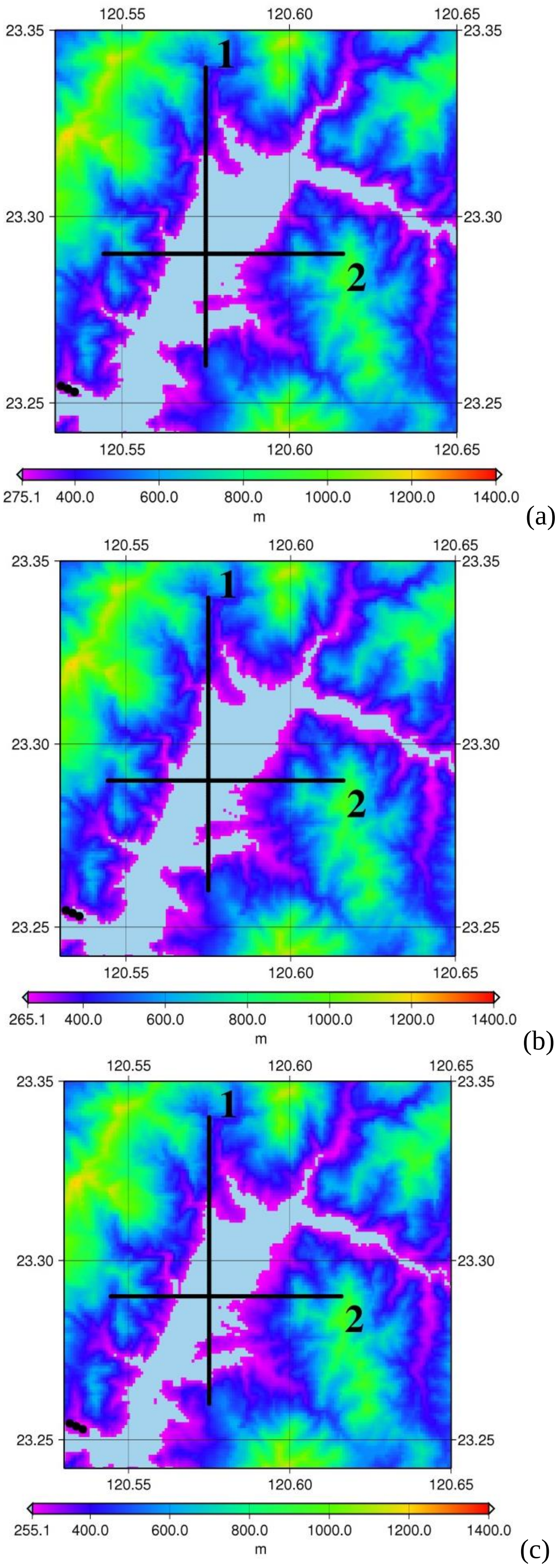

Fig. 3 DEMs with (a) high water (275 m), (b) medium water (265 m), and (c) low water (255 m) levels. The three black dots appearing in the bottom left corners mean the locations of three gravity stations. Profiles 1(north-south going) and 2 (east-west going) are also provided.

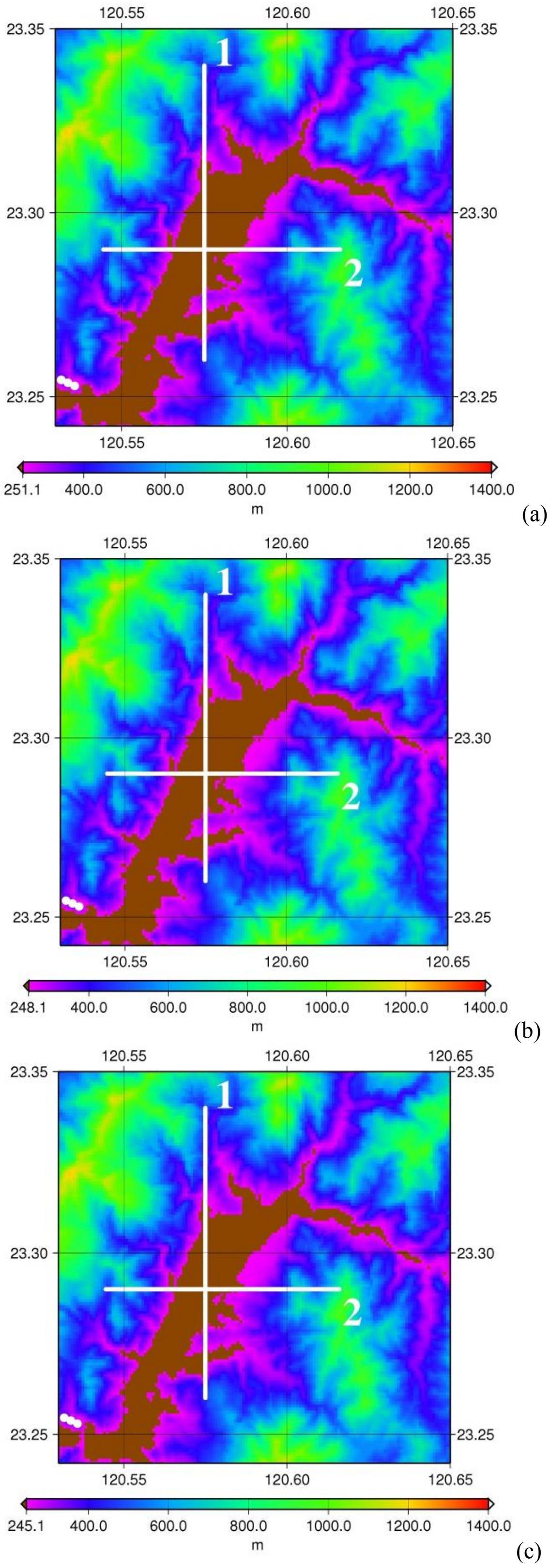

Fig. 4 DEMs with (a) high deposit (251 m), (b) medium deposit (248 m), and (c) low deposit (245 m) levels. The three gray dots appearing in the bottom left corners mean the locations of three gravity stations. Profiles 1 and 2 are also provided.

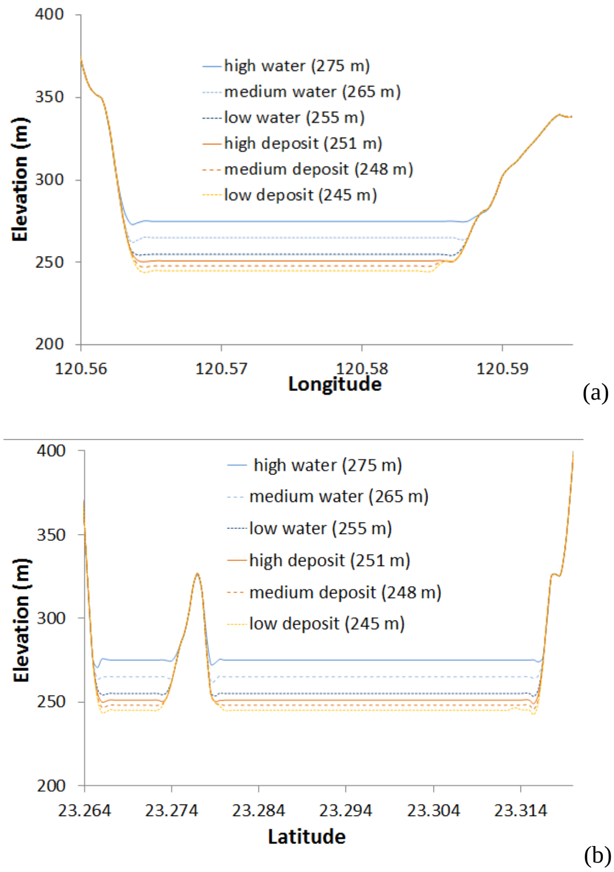

Fig. 5 The elevation variations of (a) profiles 1 and (b) 2.

3.3 Results and Discussions

The TC differences ∆TC (gravity differences) between cases 2 - 9, and case 1 at stations 1 - 3, are all summarized in **Tables 2 - 4** and **Fig. 6**. The condition of case 1 is "low water level and low deposit level." and can be regarded as the normal status of Tseng-Wen reservoir before being suffered from a natural disaster. On the contrary, Cases 2 - 9 can be considered as the statuses of the reservoirs which have suffered from heavy rains and landslides so that the amounts of water and deposits rise.

It is apparent that the TC differences (∆TC) between every 2 cases increase rapidly with increasing water and sediment volumes, and decrease gradually with increasing distance from the gravity station to reservoir. Overall, the ∆TC between every 2 cases lie on ranges from 0.014 to 0.211 mgal, from 0.014 to 0.174 mgal, and from 0.014 to 0.158 mgal at stations 1~3, respectively.

In station 1 (nearest station), 170,000 and 340,000 tons of water can lead gravity variations of 0.081 and 0.183 mgal, respectively; In station 3 (furthest station), 170,000 and 340,000 tons of water can only lead gravity variations of 0.042 and 0.092 mgal, respectively. For gravity variations of sediments in **Table 2**, 68,000 and 136,000 tons of deposits result in 0.014 and 0.028 mgal gravity

Table 2 The TC differences (ΔTC) between cases 2 - 9, and case 1 at station 1.

Case	Increasing amounts of water and sediment (tons)	ΔTC (mgal)
case2- case1	Water: 170,000 Sediment: 0	0.081
case3- case1	Water: 340,000 Sediment: 0	0.183
case4- case1	Water: 0 Sediment: 68,000	0.014
case5- case1	Water: 170,000 Sediment: 68,000	0.095
case6- case1	Water: 340,000 Sediment: 68,000	0.197
case7- case1	Water: 0 Sediment:136,000	0.028
case8- case1	Water: 170,000 Sediment: 136,000	0.109
case9- case1	Water: 340,000 Sediment:136,000	0.211

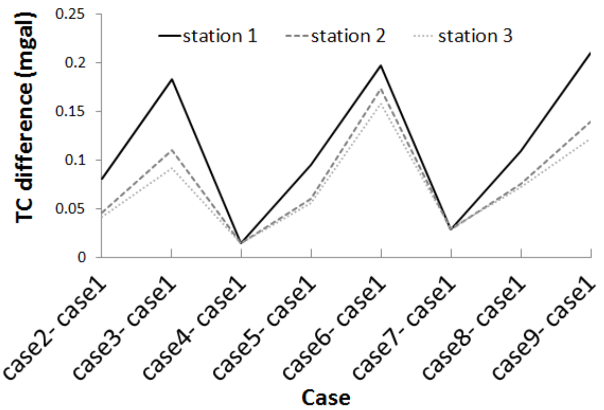

Fig. 6 The TC differences (ΔTC) between cases 2~9, and case 1 at stations 1~3.

change at station 1, and 0.014 and 0.03 mgal at station 3, respectively.

In fact, the real variation of water volume can be easily measured by a staff gauge. Therefore, water variation observed by staff gauges will be used to validate those observed by a gravimeter in future gravity surveys. If the gravity method is proven to be feasible for estimating water variation, it can be also used to estimate volume variations of reservoir sediments, which are originally difficult to be measured quickly and accurately.

In **Tables 2 - 4**, the amounts of gravity changes due to water or sediment variations are able to be observed by an absolute gravimeter (e.g., FG-5 gravimeter, resolution: 0.001 mgal), but a relative one (e.g., g-Phone gravimeter, resolution: 0.1 mgal). Therefore, an absolute gravimeter observing at

Table 3 The TC differences (ΔTC) between cases 2 - 9, and case 1 at station 2.

Case	Increasing amounts of water and sediment (tons)	ΔTC (mgal)
case2- case1	Water: 170,000 Sediment: 0	0.046
case3- case1	Water: 340,000 Sediment: 0	0.110
case4- case1	Water: 0 Sediment: 68,000	0.014
case5- case1	Water: 170,000 Sediment: 68,000	0.060
case6- case1	Water: 340,000 Sediment: 68,000	0.174
case7- case1	Water: 0 Sediment:136,000	0.029
case8- case1	Water: 170,000 Sediment: 136,000	0.075
case9- case1	Water: 340,000 Sediment:136,000	0.140

Table 4 The TC differences (ΔTC) between cases 2 - 9, and case 1 at station 3.

Case	Increasing amount of water and sediment (tons)	ΔTC (mgal)
case2- case1	Water: 170,000 Sediment: 0	0.042
case3- case1	Water: 340,000 Sediment: 0	0.092
case4- case1	Water: 0 Sediment: 68,000	0.014
case5- case1	Water: 170,000 Sediment: 68,000	0.056
case6- case1	Water: 340,000 Sediment: 68,000	0.158
case7- case1	Water: 0 Sediment:136,000	0.030
case8- case1	Water: 170,000 Sediment: 136,000	0.072
case9- case1	Water: 340,000 Sediment:136,000	0.122

nearest gravity station are more recommended for future practical gravity surveys.

Overall, the gravity changes observed at three gravity stations are obvious if a great quantity of water or sediments exists. Couples of practical experiments are being carried out during this summer and fall seasons by an FG-5 gravimeter. For these practical experiments, we will firstly use staff gauge measurements to evaluate the water volume variation estimated by an FG-5 gravimeter, and secondly use the gravity method to predict the unknown deposit variation in Tseng-Wen reservoir.

The limitation of the new method for predicting sediment volume is currently on distribution of deposit in a reservoir. In this study, we just simulate that the deposit is averagely distributed in the reservoir. The relation between deposit location and gravity variation needs to be further investigated.

4. CONCLUSIONS AND FUTURE WORKS

4.1 Conclusions

In this paper, we investigated the TC value changes (gravity changes) due to simulated reservoir conditions, including high, medium, and low water and deposit levels. The methodology of TC value computation was based on Gaussian quadrature. The principal conclusions in this study are (1) that 170,000 tons of water lead to 0.081 mgal gravity change at nearest gravity station, but only 0.042 mgal at furthest one, (2) that slight water or sediment change results in little gravity variation at three gravity stations, (3) that after the analysis of TC, absolute gravimeters are more suitable for estimating the amounts of water or sediment variations while doing practical gravity surveys.

4.2 Future works

Several practical gravity surveys around Tseng-Wen reservoir are being implemented by a FG-5 absolute gravimeter during 2014 typhoon seasons. The results will be accurately and precisely compared to the simulated results in this paper.

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