

SOIL REDUCTION FACTOR ON LIQUEFACTION POTENTIAL AREA IN TAIWAN

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ABSTRACT

The 921 Chi-Chi earthquake triggered extensive soil liquefaction in central Taiwan. The evaluation of soil liquefaction thus becomes an important topic of study. Most previous researches only performed the assessments of liquefaction potential and the method of liquefaction potential zoning. The results based on above studies focus on the evaluation of safety factors of liquefaction, the degree of danger caused by liquefaction and the damages or settlements in the liquefaction areas, but it does not assess how the results applied to engineering design. This paper presents a case history that uses reduced parameters for mat foundation design. The main objective of this research is to use the finite element method to simulate the interaction between the mat foundation and the underneath soils. It will focus on the soil reduction factor on all possible factors affecting the foundation design .

Keyword: Soil liquefaction, Soil reduction factor, Finite element

INTRODUCTION

Many domestic studies have been done so far on soil liquefaction, including geological survey, analysis of liquefaction potential, liquefaction potential zoning, design seismic force, and so on. Most of them have achieved considerable success, except in the area of anti-liquefaction engineering design where little has been done thus far. Furthermore, the domestic regulations concerning engineering design of liquefaction site refer only to subtraction of coefficient of subgrade reaction from reduction factor of soil parameters (Modulus of Subgrade Reaction). Whether the final design is considered conservative or not, it is often questionable to many and lacks substantial ground, not to mention the regular troubles it has brought to geotechnical designers.

Present evaluation methods on soil liquefaction potential are mostly applicable to the evaluation of liquefaction factor of safety, liquefaction potential index, and seismic subsidence during the analysis phase. When it comes to converting analysis results of liquefaction potential into data such as soil reduction parameters or residual strength of liquefied soil required by geotechnical designers during design phase, the available information in the literature is few and far between,

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while the related regulations only provide provisions on principles. Therefore, this study will use a case study to explore the related design process and to review basic designs that are appropriate and economical.

LITERATURE REVIEW

Generally geotechnical designers will use the methods (e.g. Seed, T-Y, NJRA, CPT- q_c) recommended by "Regulations Governing Basic Structural Design of Buildings" [10] for assessment of liquefaction potential during the analysis phase. However, the regulations do not restrict the use of specific assessment methods, which means during the analysis, designers are free to choose based on geological characteristics of the site, resulting in differences between analysis results as a result of different assessment methods used. Since the promulgation of the "Code and Commentary on Building Seismic Design" [12] by Ministry of the Interiors on July 1, 2005, according to Section 11.13 "Determining the liquefaction potential of sandy soil" which stated that "all construction sites shall consider the possibility of liquefaction occurrences separately for minor to medium earthquakes, design earthquakes, and maximum considered earthquakes". In addition, the Code takes into consideration building use factor I, which has a great impact on design outcome and project funding.

At the design stage, according to Section 10.6 in "Regulations Governing Basic Structural Design of Buildings", as to the soil that is deemed to liquefy, the design shall discount soil parameters appropriately and use it as the basis for seismic design."The code included two types of D_r table on reduction factor of soil parameter from the "Guidebook on Road-bridge Chapter V Seismic Design" of Japan Road Association (1996) and "Guidelines on Fundamental Structural Design in Buildings" of the Architectural Institute of Japan (1998); the newer edition of "Code and Commentary on Building Seismic Design" also recommended the adoption of the reduction factor as stipulated by the Japan Road Association. But there is still a substantial difference between the two types of reduction factor of soil parameters mentioned earlier, and as a result, the recommended method from the Japan Road Association is generally used in the design. This Code recommended choosing different reduction factor of soil parameters according to different anti-liquefaction shear strength ratio (R), anti-liquefaction safety factor (F_s), and corresponding depths beneath ground surface.

In practice, more conservative reduction factor is generally used in most design of public works and that increases the cost in foundation engineering; however, under economic considerations, it will be difficult for an average small-scale private project to use pile foundation or site improvement as preventive measure against soil liquefaction. This study used a four-story building with a basement as an example to demonstrate challenges facing most medium to low buildings in terms of geotechnical design, and to explore the application of reduction factor of soil parameters in the design against liquefaction.

RESEARCH METHODS

This study employed finite element analysis program SAP2000 [6] to build a foundation structure model. Using linear springs to simulate the soil beneath the foundation, with the loading of the

upper structure being transferred through point load to act on column nodes and evenly distributed load to simulate raft foundation loading, not only will it be possible to simulate the interaction between raft foundation and the soil, at the same time one can also consider the behavior of the soil springs when no tension is applied. The checklist for the analysis and design phase is described as follows:

Analysis of Liquefaction Potential

A liquefaction potential assessment was conducted on the site using methods recommended in "Building Code on Structural Design of Foundation", taking into account the provisions of Section 10.6 on "as to the soil that is deemed to liquefy, the design shall discount soil parameters appropriately and use it as the basis for seismic design". This study used the NJRA liquefaction potential assessment methods and the reduction factor of soil parameters recommended by Japan Road Association as parameters in foundation design, and followed the anti-liquefaction safety factor FL derived from liquefaction assessment results to determine the reduction factor of soil parameter. The so-called soil parameters in this section refers to the "coefficient of subgrade reaction described in Section 11.14 of the Code". Therefore, this study will discount the reduction factor of soil parameters by vertical coefficient of subgrade reaction to simulate the soil springs, analyzed with structural analysis models, and then compare the extent of impact that coefficient of subgrade reaction has on the foundation structure before and during liquefaction.

For the ground that showed liquefaction potential in site liquefaction potential analysis, initial cross-section dimensions and deformation amount on each node were collected by entering the design parameters unamended for reduction factor of soil parameters into the structural model for analysis. Next, the design parameters amended for reduction factor of soil parameters were entered into the structural model for analysis in order to work out the degree of settlement (lifting) and bending on each node by varying plate thickness in the calculation. On the condition that the settlement (deformation) and angular distortion are within the allowable limit stipulated by "Building Code on Structural Design of Foundation" and there is no structural damage, compare the discounted design parameters and the undiscounted design parameters, and explore the differences between them.

Vertical Coefficient of Subgrade Reaction of Soil

Vertical coefficient of subgrade reaction of soil refers to the ratio between loading stress of each unit area and soil deformation when loading is applied within the elastic range. Based on the site drilling data, the K_v prior to liquefaction of the soil beneath the foundation should be used as coefficient of subgrade reaction for sandy soil, as shown in Table 1.

Table 1 Vertical coefficient of subgrade reaction of soil

GL	Vertical coefficient of subgrade reaction of soil (t/m³)
-6.0 ~ -11.65m	640
-11.65 ~ 23.45m	1310
-23.45 ~ -29.05m	1120
-29.05 ~ -31.60m	1810

After considering the effect of the thickness of liquefied soil, studied by Xie et al (2005) [14], formula for foundation settlement assessment and definition of vertical coefficient of subgrade reaction proposed by Das (1990) [2], this study suggests to first discounting soil elastic modulus in order to achieve the objective of reduced coefficient of subgrade reaction. The discounting method is as follows:

$$S = C_1 C_2 \Delta q \sum_0^{2B} \frac{I_z}{E_s} \Delta Z \quad (1)$$

$$K_v = \frac{\Delta q}{S} \quad (2)$$

Where:

S : foundation settlement

C_1 : foundation buried depth correction factor

C_2 : correction factor of time effect

Δq : net pressure increase acting on the underside of the foundation

I_z : Strain influence factor

E_s : Young's modulus of the soil

B : foundation width

ΔZ : is the thickness of soil layers within the depth of $2B$ beneath the foundation, and adding (1) and (2) will produce :

$$\frac{1}{K_v} = C_1 C_2 \sum_0^{2B} \frac{I_z}{E_s} \Delta Z \quad (3)$$

when liquefied, use D_e to deduct from Young's modulus of liquefied soil

$$E_s^* = E_s \times D_e \Rightarrow \frac{1}{K_v^*} = C_1 C_2 \sum_0^{2B} \frac{I_z}{E_s^*} \Delta Z \quad (4)$$

By formula (3) and (4):

$$K_v^* = K_v \times \frac{\sum_0^{2B} \frac{I_z}{E_s} \Delta Z}{\sum_0^{2B} \frac{I_z}{E_s^*} \Delta Z} = K_v \times D_e^* \quad (5)$$

D_e is the reduction factor of soil parameters after discounting Young's modulus of soil. Based on the method of discounting Yang's modulus of soil proposed by Xie et al [14], this study also took into consideration factors such as foundation size and thickness of liquefied soil on top of the method of direct deduction against coefficient of subgrade reaction recommended by aforementioned Code.

Foundation Support and Settlement

Based on the original construction plan, the designed foundation was raft foundation, with a dimension of 30m×57m and a depth of approximately GL.-5.5m. According to geological data of the site, the foundation is located on the second layer of medium stiff clay soil, with groundwater level at GL.-2.5m, thus the foundation support is primarily controlled by the second to the fifth layer of the soil. According to Building Code on Structural Design of Foundation, bearing capacity is analyzed using the following formula:

$$q_u = c N_c F_{cs} F_{cd} F_{ci} + \gamma_2 D_f N_q F_{qs} F_{qd} F_{qi} + 0.5 \gamma_1 B N_r F_{rs} F_{rd} F_{ri} \quad (6)$$

$$q_{all} = \frac{q_u - \gamma_2 D_f}{FS} + \gamma_1 D_f \quad (7)$$

in formula

q_u : ultimate bearing strength (t/m^2)

C : cohesion of soil beneath the foundation slab (t/m^2)

γ_1 : average unit weight of soil within depth B beneath foundation slab, and if below groundwater level, it shall be the soil's effective unit weight (t/m^3)

γ_2 : average unit weight of soil above foundation slab, and if below groundwater level, it shall be the soil's effective unit weight (t/m^3)

D_f : the distance between the lower ground level near foundation and the underside of the foundation slab (m)

B : width of a rectangular footing (m)

L : length of a rectangular footing (m)

N_c, N_q, N_r : bearing strength factor

F_{cs}, F_{qs}, F_{rs} : form factors

F_{cd}, F_{qd}, F_{rd} : buried depth factors

F_{ci}, F_{qi}, F_{ri} : load inclination factors

FS : safety factor, which must not be less than 3 for long-term bearing capacity in a building. When considering short-term load such as earthquakes, wind, and snow, the allowable bearing capacity must be raised by 50%.

As the soil under the foundation is not homogeneous (alternating thin and interbedded layers), for $c-\phi$, the shear strength parameters of this soil with alternating thin and interbedded layers were calculated based on the weighted average method proposed by Bowels [1]; for soil within depth B under the foundation slab, the study considered deducting shear strength parameters of the soil from the reduction factor of soil parameters.

Analysis of Foundation Settlement

In foundation design, apart from considering the load factors, one must also pay attention to the foundation's total amount of settlement and ensure it does not exceed the allowable limit, to avoid damage to the structure by excessive or uneven settlement. There are many factors that determine the amount of settlement in a structures, for example, the type of structure, its size, location, use, and rate of settlement, etc; generally uneven settlement is most damaging to a structure, and uneven settlement can also be described using angular distortion. The discussion of foundation settlement usually includes the consideration of total amount of compression caused by elastic deformation of soil and phenomenon of compaction, that is the total amount of settlement between instantaneous settlement and compaction settlement. Therefore, a designer must look at building type and carefully assess any settlement that may arise. Except for architectural aesthetic reasons or special structural requirements, the generally permitted amount of angular distortion due to foundation settlement and total settlement is as the following:

1. The allowed angular distortion: any angular distortion between two adjacent columns or points caused by differential settlement shall not lead to the formation of dangerous cracks in the building or affect negatively the use of the building. The relationship between angular distortion and extent of damage in building is as shown in Table 2.

2. The allowed amount of settlement: in principle, the total amount of settlement in a building caused by foundation loading must not exceed the values shown in Table 3 (commentary in Section 4.4.8 of Building Code on Structural Design of Foundation), but one should pay attention to the actual condition of the structure, as minor settlement can also lead to damage. In a case where the settlement amount of the building exceeded the allowed limits above, appropriate measures such as increasing the material strength of the structure, alter the form of the structure, or improve the ground condition, must be taken to prevent damage to the structure by settlement.

Table 2 The relationship between angular distortion and extent of damage in building

Angular Distortion	Extent of damage in building
1 / 600	Risk of damage to the bracing structure
1 / 500	Safety margin (including safety factor) for disallowing cracks in the building
1 / 300	Formation of cracks on partition walls (excluding safety factor)
1 / 250	Tilting in rigid high-rise buildings becomes visible
1 / 150	Extensive amount of cracks on partition walls and brick walls
1 / 125	Safety margin of flexible brick walls (including safety factor)

Table 3 Foundation types and allowable settlement amount (cm)

Type of structure	Concrete	Reinforced concrete		
Type of foundation	Continuous footing	Single column and combined foundation	Continuous footing	Raft foundation
Total amount of settlement	4.0	10.0	20.0	30.0

According to the provisions in Section 4.4 of Building Code on Structural Design of Foundation, the vertical stress increment σ_z at any underground point caused by the downforce of the foundation slab is:

Vertical stress increment σ_z as follows:

$$\sigma_z = \frac{3PZ^3}{2\pi R^5} \quad (8)$$

and the amount of compaction settlement H_c is:

$$H_c = \sum_{i=1}^n \frac{e_{oi} - e_{fi}}{1 + e_{oi}} H_i \quad (9)$$

in formula:

σ_z : vertical stress increment at depth Z in the ground (t/m^2)

P : Vertical concentrated load of the underside of the foundation (t)

Z : depth of calculation point Z below the underside of the foundation (m)

R : distance to point of concentrated loading (m)

H_c : the amount of settlement (cm)

e_{oi} : the initial positive stress of layer i in the soil pore ratio at σ_{oi}

e_{fi} : the initial positive stress of layer i in the soil pore ratio at σ_{fi}

σ_{oi} : the initial positive and effective vertical stress of the center point at layer i

σ_{fi} : the final positive and effective vertical stress of the center point at layer i

H_i : layer i thickness (cm)

CASE ANALYSIS AND STUDY

Project Overview and Geological Conditions of the Site

The case used in this study is a new public building with four floors above ground and one level of basement, located in Puli, Nantou County. On site, a total of six boreholes were installed, of which BH1 was about 35 meters deep, BH2, BH3, and BH4 were about 25 meters deep, and BH5, BH6 were about 15 meters deep. The ground formation of the site can be roughly divided into five layers from the top to the bottom, and please refer to Table 4 Simplified Soil Parameters for details.

Based on the measurements taken during drilling, the groundwater level is approximately 2.5 meters below the surface. For safety reasons, this study had used -1.5m as the long-term groundwater level.

Table 4 Soil Simplified Soil Parameters

Depth	Description of geological conditions	N	ω %	γ_i t/m ³	S_u t/m ²	C t/m ²	ϕ	C_c	e	E_s
First layer 0~4.8m	Yellow-brown or gray fine sand mixed with sandy silt	7	21.79	1.97	-	0	30	-	0.66	980
Second layer 4.8 ~ 10.8m	Yellow-brown silt Clay	8	29.2	1.93	3.5	-	-	0.195	0.83	1500
3rd layer 10.8m ~ 22.6m	Gray silty fine sand inter-bedded with sandy silt, with occasional then clay layer	17	24.35	1.96	-	0	33	-	0.70	2380
4th layer 22.6m ~ 28.2m	Gray silty clay Soil	14	25.66	1.96	7.0	-	-	0.263	0.75	2620
Fifth layer 28.2m ~ 37.0m	Gray clayey silt with fine sand	29	25.35	1.99	-	0	33	-	0.71	4060

Liquefaction Analysis

The study conducted liquefaction potential assessment on each borehole in accordance with the latest "Building Regulations and Commentary on Seismic Design", Section 11.13 "Determining the liquefaction potential of sandy soil" which stated that "all construction sites shall consider the possibility of liquefaction occurrences separately for minor to medium earthquakes, design earthquakes, and maximum considered earthquakes".

The regulations stipulated that the short-term design spectral acceleration for this site is $S_s^D = 0.7$; maximum considered spectral acceleration coefficient is $S_s^M = 0.9$; 1.5 was used as use factor I, In accordance with the regulations, the study then checked for the possibility of liquefaction occurrence for the three different situations ($S_{DS} = F_a S_s^D$; $S_{MS} = F_a S_s^M$, the site is classified as Category II construction site $F_a = 1.0$), the assessment results are described as follows:

1. During minor to moderate earthquakes: A is used as the surface horizontal acceleration for general sites and sites near a fault $A = \frac{0.4 S_{DS} I}{4.2} g = 0.1g$. Based on the decision of long-term groundwater table at 1.5 m below the surface, the study found that the anti-liquefaction strength of the soil on site was greater than 1.0. According to the liquefaction potential index proposed by Iwasaki et al

(1982) [3] for the assessment of soil liquefaction severity, the assessment result $P_L=0$, which indicated no possibility of liquefaction. So when minor to moderate earthquakes are considered, there is no sign of liquefaction in the sandy soil layer below the foundation, which means the current raft foundation used in building foundation planning should be able to meet the design safety requirements.

2. During design earthquakes: surface horizontal acceleration $A=0.4S_{ds}I_g=0.42g$ was used. Based on the decision of long-term groundwater table at 1.5 meters below the surface, the study found that the anti-liquefaction strength of the soil on site to be between 0.4 and 1.0, with liquefaction potential index P_L between 4.35 ~ 15.67. Apart from the serious liquefaction found in BH2, the rest showed mild to moderate liquefaction, with liquefaction potential index averaged 11.10.

3. During maximum earthquake consideration: surface horizontal acceleration $A=0.4S_{ms}I_g=0.54g$ was used, and based on the decision of long-term groundwater table at 1.5 meters below the surface, the study found that the anti-liquefaction strength of the soil on site was between 0.1 ~ 1.0, with liquefaction potential index P_L between 6.25 ~ 22.64. It is classified as moderate to severe liquefaction and the liquefaction potential index averaged 18.2.

Analysis of the Foundation Bearing Forces

According to the analysis of formula (6) and (7), the allowed bearing capacity of the foundation prior to liquefaction as $q_a = 25.6 \text{ t/m}^2$, and the short-term allowed bearing capacity as $q_{ac} = 38.4 \text{ t/m}^2$; if the reduction factor of soil parameters for sandy soil is considered during design earthquake, then during liquefaction, the allowed bearing force of the soil under the foundation is $q_a = 22.5 \text{ t/m}^2$, and the short-term allowed bearing capacity is $q_{ac} = 33.8 \text{ t/m}^2$. As this site's allowable bearing capacity before or during the soil liquefaction are both greater than the maximum load of the structure, under the condition of even loading of the foundation, the bearing capacity of the ground under the present foundation will meet the legal requirements irrespective of soil liquefaction.

When uneven loading of the foundation was considered, contact pressure of the foundation was approximately 28.4 t/m^2 , and the allowable bearing capacities of the soil under the foundation before or during liquefaction were all smaller than the uneven load of the foundation. It was clear that the allowable bearing capacities were inadequate. Therefore, consideration should be given to improve parts of the ground at the site, in order to increase the soil's bearing capacities and to ensure the safety of the upper structure.

Settlement Analysis

The excavation depth of the site was about 5.5m, and the effective vertical soil pressure at that depth was 7.2 t/m^2 . Based preliminary analysis of the architectural planning data, the building's static and live load were calculated, which were then used to analyze the amount of settlement in the foundation. The calculation results are discussed as follows:

Assuming the raft foundation is fully waterproof and the usual water buoyancy of 3.4 t/m^2 , the maximum amount of settlement calculated was 6.85 cm, the maximum amount of uplift was 0.85 cm, and the maximum degree of deflection was $1/130$. They complied with the requirements on

raft foundation in the Building Code on Structural Design of Foundation, which stipulated that "the maximum amount of settlement (uplift) must not exceed 30cm", but did not meet the requirement of maximum deflection less than $1/250$.

Finite Element Analysis

The above analysis shows that, when considering design earthquake and maximum earthquake consideration, the local strata on the construction site has higher liquefaction potential, which means according to the regulations, it is necessary to overcome the liquefaction issue and check for safety after liquefaction occurred by way of appropriate foundation design or site improvement. Therefore, with the aid of SAP2000 finite element software and the reference to recommendations made by Xie et al [14], as well as reduction formula that incorporate the thickness of liquefied soil and foundation size, the study took modulus of subgrade reaction with and without reduction, and entered them separately into the structural model for analysis. Calculations on cross-section of different slab thickness were also done to explore its relevance.

Analysis showed that before and during liquefaction, the differential settlement of the raft foundation decreased as the slab thickness increased, but it increased with the reduction of modulus of subgrade reaction. The higher the reduction, the greater the significance of the effect that slab thickness has on differential settlement (or angular distortion), that is, the effect that altered slab thickness has on the reduction in differential settlement is more significant when vertical subgrade coefficient is smaller. Therefore, the main research direction of this study is to use finite element analysis to alter the size or type of the structure, allowing the structure to settle within the allowable differential settlement limits of the foundation so that the structure is legally compliant and economically efficient. Due to the use of use factor $I=1.5$ for the case, the design earthquake and maximum earthquake consideration have also increased by 1.5 times, which resulted in higher liquefaction potential figures for the site during the liquefaction analysis. Although the option of changing structural size and type was considered during the analysis to reduce its differential settlement, the final decision taken was to increase the structural size considerably in order to meet the regulatory requirements on angular distortion. This was not economically efficient. To ensure the safety of the upper structure, the study recommended site improvements to the sandy soil areas that have higher liquefaction potential, and minor site improvements to the clayey soil areas.

In addition, this study had also taken into account modulus of subgrade reaction recommended by various scholars in its assessment, only these recommended assessment values from scholars such as

Terzaghi (1955) [4], Vesic (1961) [5], Bowels (1988) [1], and Xie et al (1996) [11] do vary quite substantially as high as 10 times, which had a huge impact on the analysis results.

CONCLUSIONS AND RECOMMENDATIONS

According to the latest Building Regulations and Commentary on Seismic Design, the possibility of liquefaction occurrence is not permitted on a construction site during a minor or moderate earthquake, and the anti-liquefaction rate must not be less than 1.0 (if liquefaction is likely under

this condition, improvements must be made to the site); during design earthquake and maximum earthquake consideration, soil liquefaction is allowed but the appropriate foundation type should be used for the building. Furthermore, reduction to soil parameters of the seismic design must be done according to actual conditions, and safety of the building and the foundation after ground liquefaction has occurred on site must be checked. There are still a number of points worth debating concerning the practical application of reduction factor of soil parameters stipulated in the regulations:

1. The liquefaction safety factor F_L used in assessment D_ε does not take into account the thickness of liquefied soil, i.e the D_ε values are all the same for liquefied layers of different thickness.
2. The regulations contained no relevant provisions on how to estimate the D_ε value of interbedded soil.
3. D_ε value did not take into account the effect of interaction between foundation structural size and the soil. This study adopted suggestions made by Xie et al [14] and took into consideration the reduction formula on thickness of liquefied soil and foundation size. In practice, this will not only help in solving some of the above concerns, but also provides a direction of reference for the designers.

This study revealed that, in terms of foundation planning and design for areas with liquefaction potential in medium to low multi-story buildings and buildings of public use, in addition to the use of pile foundation, not only can one consider local geological improvements when budget constraints is an issue, but also the method of designing the parameters according to the reduction recommended by the regulations and allowing the structure to settle, then through structural analysis to find the ground that meets the regulations. This construction method should be of a more economical one.

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