BACK-ANALYSIS OF APPARENT BASAL FRICTION OF TSAOLING AND JIUFENERSHAN LANDSLIDES, 1999

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ABSTRACT

Simulation of rock avalanches using continuum models has been developed and verified as a useful tool for understanding the kinetic properties of real avalanches. The model equations with this approach share great similarities to shallow water equations. The friction/shear forces exerted on the flow by the basal surface are the most important factor influencing the flow run-out distances. For validation of the friction effects and characterization of real landslides, we back analyze the parameters of the Coulomb friction and Voellmy rheology laws for two major landslide events, triggered by the Chi-Chi earthquake in 1999: Tsaoling and Jiufenershan landslides. The best fit of the flow parameters are determined by minimizing the standard deviation between the simulation and the field measurements. In addition, the kinetic properties such as the landslide duration and velocity are concluded.

Key Words: Coulomb friction, Voellmy rheology, Chi-Chi earthquake, Tsaoling landslide, Jiufenershan landslide

INTRODUCTION

Geological and climate conditions of Taiwan are the major causes of rapid morphological changes: the frequent earthquakes, subtropical typhoons. Consequently, landslides and debris flows are often responsible for the fast shaping processes of the morphology. For example, at the time of writing, Taiwan is suffering from the strike of Morakot typhoon and Siaolin Village, of Kaohsiung County, is buried after a catastrophic debris avalanche event. In the present paper, we perform the back analysis on the parameters of two fundamental rheological laws for two major rock avalanches trigged by the Chi-Chi earthquake, 1999. The rheology laws are the simple Coulomb and Voellmy basal friction laws and the landslides are Tsaoling and Jiufenershan landslides.

Rheology laws are in general referred to the stress and strain relations when associated with continuum mechanics, but herein we refer the term to the friction laws on the fixed basal surface, which is the closure relation to the model equations. The model equations from the continuum mechanics share great similarities to shallow water equations which have long been widely used in the hydrology engineering. Milestone extensions equipping such kind of models with capabilities handling general topographies include Bouchut and Westdickenberg

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(2004), Luca *et al.* (2009), who introduce terrain-fitted coordinates, and Tai and Kuo (2008), who further incorporate temporal variable basal surfaces, and other references documented therein. While efforts have been continuously being made with the rheological and frictional laws in small scale laboratory tests, e.g. Tai and Lin (2008), we need benchmark results of the continuum model for real landslides to validate the applicability to the real environment. Using the Eulerian model with the simple Coulomb friction law, we applied the model to Tsaoling landslide. The kinetic properties such as the landslide duration and velocity were resolved and concluded. One of the major conclusions is that the sole rheological parameter, the Coulomb friction angle, is regressed to 6 degrees, Kuo *et al.* (2009).

As more sophisticated rheological models are proposed, the general tendency is that more physical influential parameters are taken into considerations. This makes the back analysis for the rheological parameters with real landslide events important. Similar work has been performed by many prior investigators. For example, Hungr and Evans (1996) concluded that Voellmy friction law is appropriate for a set of 23 landslides using a pseudo one-dimensional dynamic model. Hungr and McDougall (2005) analyzed Noamsh River landslide 1999 with Voellmy rheology, and Pirulli and Mangeney (2008) verified Frank, Canada 1903, and Val Pola, Italy 1987, landslides with Coulomb, Voellmy, as well as Pouliquen friction rheology, Pouliquen and Forterre (2002). Conclusions have been made on the parameters among a set of different friction models.

The best sets of parameters in the preceding research are determined by two different proposals: by the best fit of the outlines of deposits or by the best fit of the height/run-out distance on the one-dimensional main landslide profiles. However, for the former, the outlines of deposits depend sensitively on the choice of the cut-off depth with the shallow water types of models and, for the later, the choice of the main profile remains somewhat arbitrary. Therefore, in the present paper, we propose to determine the parameter sets by minimizing the standard deviation between the simulation and the field measurement over the deposit area. This scheme is the same as in Kuo *et al.* (2009). This eliminates the need to subjectively inspect the simulation results and enables us to automate the regression processes by any suitable optimization schemes.

We choose the simplex method to obtain the parameter sets for the landslides. In the following sections, we first describe the continuum shallow water model, the optimization object function, brief summaries of the landslides and the results of the back analysis.

MODEL EQUATIONS

To simulate the landslide motion, detailed composition and the initiation mechanisms of the geological materials have to be neglected. Under these simplifications, we omit the failure of the interface under the earthquake motion, rock rupture, and volume dilation of the mass. We assume the process takes place in a relatively short time (compared to the total duration) after the landslide is triggered. The flow materials are then assumed a single incompressible constituent and, when at motion, the flow has a uniform velocity across the flow depth.

The rheology laws in this paper are referred to the basal friction laws, which include the Coulomb and the Voellmy models. The basal friction is the most important factor to influence the avalanche motion according to the analysis of Luca *et al.* (2009). Gray *et al.* (2003) confirm that by neglecting the fluid constitutive details, acceptable quantitative results

compared to laboratory tests can still be obtained. With considerations for general topography, the governing equations of the frictional Eulerian fluid model read

$$\partial_t h + c\nabla_x \cdot (h\mathbf{u}) = 0, \tag{1}$$

$$\partial_t (h\mathbf{u}) + c\nabla_x \cdot (h\mathbf{u} \otimes \mathbf{u}) = -\frac{h}{c}(I - \mathbf{s} \otimes \mathbf{s})\nabla_x (g(hc+b))$$

$$-\frac{h}{c}(\mathbf{u}^t H\mathbf{u})s + \frac{h}{c}(\mathbf{s}^t H\mathbf{u})\mathbf{u} - \frac{\mu g h \mathbf{u}}{\sqrt{c^2 |\mathbf{u}|^2 + (\mathbf{s}\cdot\mathbf{u})^2}}\left(1 + \frac{\mathbf{u}^t H\mathbf{u}}{gc}\right), \tag{2}$$

where the first equation is the mass conservation equation and the second is the momentum conservation. The physical variables are the flow depth, h, defined in the normal direction to the basal surface, and the two-component velocity parametrization, $\mathbf{u}$, such that the three-dimensional Cartesian velocity, $\vec{v}$, is $\vec{v} = (c\mathbf{u}, \mathbf{s}\cdot\mathbf{u})$. Symbols c and $\mathbf{s}$ are defined by the basal topography $b = b(x, y)$, and the normal unit vector of the basal topography $\vec{n} = (-\nabla_x b, 1)/\sqrt{1 + |\nabla_x b|^2} \equiv (-\mathbf{s}, c) \in R^2 \times R^1$, where $\nabla_x = e_x \frac{\partial}{\partial x} + e_y \frac{\partial}{\partial y}$. The topographical curvature is denoted $H = c^3$旄$_x$ $_x b$. The resulting system of governing equations is of hyperbolic type. Equations of this type allow the formation of discontinuities and weak solutions (shocks).

The momentum equation contains a series of sources on the right hand side of (2). The first term is the hydraulic pressure and the gravity force, the second and third term are the sources due to the topographical curvature, where the former has the form of the centrifugal force of the flow and the latter arises due to the particular choice of the velocity parametrization.

The basal friction force is the last term of (2). The friction force, which is tangential to the topography surface in the opposite direction to the flow velocity, is assumed scaled to the normal pressure exerted on the basal surface with a friction coefficient μ. For the Coulomb friction law, the friction coefficient is related to the friction angle, ϕ, by

$$\mu = \tan\phi . \tag{3}$$

With the Coulomb rheology, ϕ is assumed a constant during the flow at motion. For the Voellmy rheology, Voellmy (1955), the coefficient contains two physical factors: the static Coulomb friction, similar to the previous simple friction law, and the second turbulent related term

$$\mu = (1 + \frac{\vec{v}^2}{\zeta h})\tan\phi . \tag{4}$$

The additional parameter, ζ, with a dimension of acceleration, is related to the turbulent motion in the flow. This friction law reflects that the strength of a granular material under rapid sharing increases with the square of the strain rate, hence, with the square of flow velocity, Hungr and Evans (1996).

The model equations are numerically solved by a shock capturing finite volume scheme based on Suliciu's approximate Riemann solver. Source terms are treated efficiently by employing a well-balancing technique, which ensures the property of preserving steady states at the

discrete level. Details of theoretical derivation are referred to Bouchut and Westdickenberg (2004), and validations of the numerical models are referenced to Mangeney-Castelnau *et al.* (2005), Mangeney *et al.* (2007), and Kuo *et al.* (2009).

MULTIDIMENSIONAL MINIMIZATION FOR FRICTION LAWS

Instead of heuristic determination of rheological parameters, an optimization process is proposed in the present paper. The process is to minimize an object function over the rheological parameter domain (ϕ, ζ). We define the object function to be the square of the difference between the simulation and measurement over the deposit area and, therefore, the scheme reads

$$h_{std}^2(\phi,\zeta) = \min_{\phi,\varsigma}\left\{\frac{1}{A}\int_A (h(\phi,\zeta)-h_{meas})^2 \mathrm{d}A\right\}, \tag{5}$$

where h_{std} is the standard deviation of the deposit depth, h_{meas} is the measured deposition depth, and $h(\phi,\zeta)$ is the simulated depth with the rheological parameters ϕ and ζ (if Voellmy law). The simulation time is set long enough to ensure at the end of simulation, the flow is nearly at rest. Illustration of the numerical phenomena of the two landslides will be given in the next section. The deposition area is A, which is chosen somewhat larger than the union area of the simulated and measured deposits. Because outside the deposit area, the integrand reduces to zero or at most at the numerical vacuum round-off error, the oversized area does not influence the optimization result.

We use the simplex method, Nelder and Mead (1965), to minimize the objective function in a many-dimensional space. Once it converges to a set of rheological parameter set, we scale the deposit volume to the measurement to determine the volume dilation ratio. The volume dilation can, of course, be promoted to be a variable of the optimization scheme, which becomes one extra dimension for optimization, but, however, this only alters the result with a negligible amount.

The proposed optimization process eliminates the need to subjectively inspect the simulation results and enables us to automate the regression processes by any suitable optimization schemes.

APPLICATION TO THE LANDSLIDES

Both of the investigated landslides were triggered by the Chi-Chi earthquake in 1999 and, from the field observation, they are both of rock avalanche type. The scar, deposit and surrounding topography and the main profiles of geological concerns are sketched in Fig. 1. The geological details of the two landslides are documented in Hung *et al.* (2000), Chang *et al.* (2005), Wu *et al.* (2005), Chen *et al.* (2006), and Kuo *et al.* (2009), and the overviews of the two landslides are as follows.

(a) Tsaoling landslide:

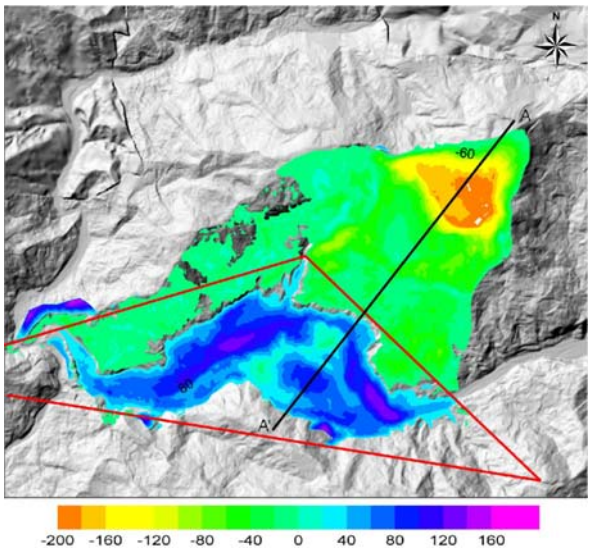
The digital elevation map (DEM) are taken from 1989 40×40m and 1999 10×10m accuracy aerial photographs. The scar volume is 126×10^6 m^3 and the deposit dilates to

150×10^6 m^3. The traveling distance of the flow is about 1.9 km with 500 m descent in elevation. The sliding surface has an average inclination angle 15°. It is concluded from the field observation that this landslide is of rock avalanche type. The flow is mostly composed by the materials of Cholan formation, mainly shale and sandstone. Its density is about 2,600 kg/m^3 and the peak and residue internal friction angles of the shale (the weaker composite) is 20.5° and 21.5°, respectively.

(b) Jiufenershan landslide:

We use the 1989 and 1999 10×10 m aerial photograph DEMs. The scar volume is 42×10^6 m^3 and the deposit is 50×10^6 m^3. The traveling distance is about 0.9 km with 250 m descent in elevation. On the main profile, the landslide mass is composed of a roughly 60 m thick, 1.5 km long sedimentary pile of shale and sandstones. The inclination angle is about 22°. By the geological observation, this landslide is flexural buckling rupture at the foothill of the slope. The avalanche material has an average density about 2,550 kg/m^3 and the internal friction angle is about 28°.

(a)



(b)

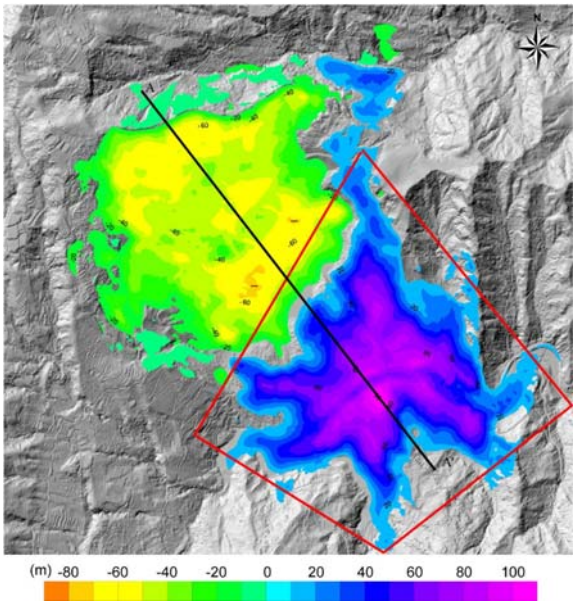


Fig. 1 Isopach map of the landslides (a) Tsaoling, (b) Jiufenershan. The regions framed by the bold red lines are

the areas for the back-analysis minimization procedure. Lines AA' define the main profiles.

The optimization scheme (3) is applied with the two rheology models for the two landslides. The deposit areas for the optimization scheme are enclosed by the bold red outlines in Figs. 1(a) and 1(b). Starting with the Coulomb friction, the initial friction angle is set to a few degrees less than the average inclination angle. For each set of parameters, we use fixed meshes for simulation in each iteration: they are 15×15 m for Tsaoling and 10×10 m for Jiufenershan. When the optimum friction angles are obtained, the additional Veollmy turbulent parameter is added into the optimization scheme, with a reasonable large value for the first iteration. Convergent criterion is set a hundredth (1/100) between successive iterations. A few different sets of initial data are tested and they converge stably to the parameters reported in the present paper. The optimization results and the flow characteristics are summarized in Table 1.

The flow kinematical properties are determined by the average mass fluxes and flow velocities on the main profiles. Typical examples of the two physical quantities versus the simulation time are plotted in Fig. 2. Their rheological parameters are taken from the optimal sets of the two friction laws. The figures demonstrate that there are two flow regimes: the main flowing cycle and the small diffusive motion afterwards. To ensure good representative deposit profiles, we set the simulation time long enough such that the fluxes at the end of simulation is less than about the twentieth (1/20) of the flux maxima. This flow duration is also determined by this criterion.

The influence of the rheology models can also be seen in this figure. Interestingly, for the two landslides, the Voellmy rheology plays different roles. While it has almost negligible effects on Tsaoling site, it delays the avalanche development significantly on Jiufenershan. For the latter, the maximum flux is lessened by 18% and the flow duration is consequently extended from 77 sec to 85 sec. This is because the dissipation of the turbulence suppresses the peak flux. We suspect that the role alternation of the turbulent dissipation is because the Voellmy coefficient ζ has a dimension of acceleration which, in principal, scales with the characteristic length of landslides. In addition, Jiufenershan has high water content in contrast to Tsaoling. It is however yet to tell an explicit relation between the coefficient and the landslide volume/run-out distances with the current data. On the other hand, comparing to the landslides with similar sliding volumes, we find that the Voellmy rheology parameters of Jiufenershan agree to the documented data, Hungr and Evans (1996), within an order of magnitude.

The final deposits are shown in Fig. 3. For both rheology models, the deposits tend to be smooth on the surface. Noticeably, the deposit hill of Tsaoling landslide, Taochiashan in the opposite of Chinshui River, is not shown in the simulation. The formation of the deposit hill is due to the reflected surge produced by the avalanche flow impinging on the slope opposite to the river valley, Kuo *et al.* (2009). The single layer of the Coulomb friction law can no longer accurately model the complicated flow interactions and the constant friction angle is too small to stop the surge from propagating upstream and smearing the deposit. The agreement, on the other hand, is good with Jiufenershan landslide because the flow does not exhibit surge interactions as in the Tsaoling case.

It is well known that the apparent basal friction angle is inversely scaled with the volume of landslides. Legros (2002) argues and extends the friction angle versus the representative

granular size and the flow volume based on discrete element simulations, Legros (2002), sketched in Fig. 4. In the shaded area, he projected the functional relation among the three factors for natural landslides, as their granular size ranges from a few centimeters to a few meters. Although it is a qualitative inference, we found the optimized basal friction angles of the two landslides fall reasonably within the expected area, just somewhat biased from the simulation curve. Of course, they do not yet reach any statistically significant conclusion, but the correspondence is supportive both to the phenomenal study and to the back-analysis of the landslides. In natural landslides, unlike the uni-size spherical granular flows in simulation, the particle size is dispersed with irregular angularities. To achieve more precise relation for the friction angle, we rely on compiling more practical cases.

Table 1 Rheology parameters and flow characteristics of Tsaoling and Jiufenershan landslides

	Coulomb Friction angle				**Voellmy Rheology**				
	ϕ	volume dilation	h_{std}	Flowing time	ϕ	ζ	Volume dilation	h_{std}	Flowing time
Tsaoling	6.9°	1.26	22.0	80 sec	6.9°	69000	1.27	23.7	80 sec
Jiufenershan	11.0°	1.22	14.7	77 sec	10.1°	91.7	1.23	15.9	85 sec

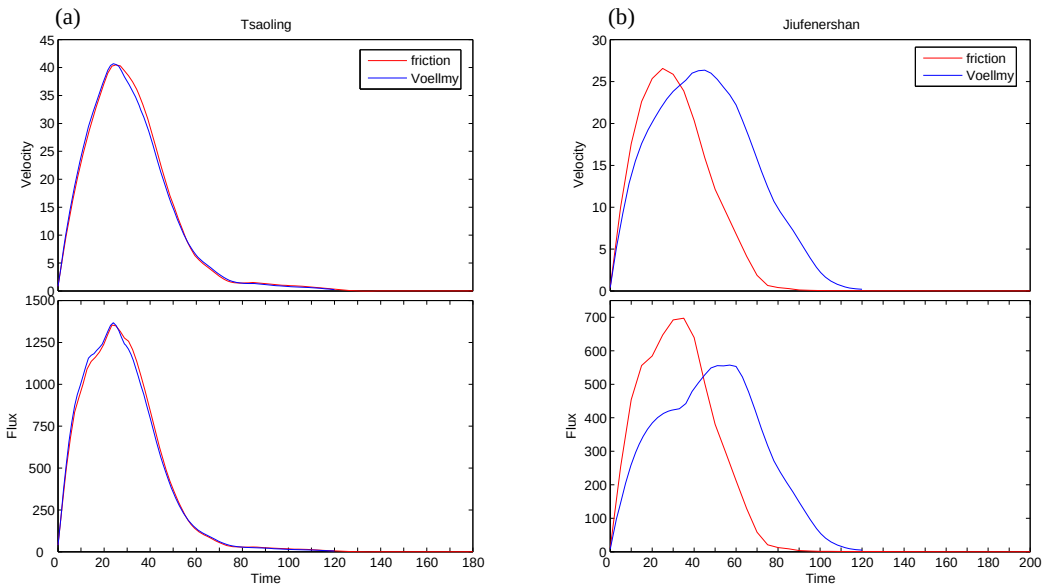


Fig. 2 The average fluxes and velocities versus time on the main profiles. Rheological parameters are taken from Table 1. (a) Tsaoling and (b) Jiufenershan.

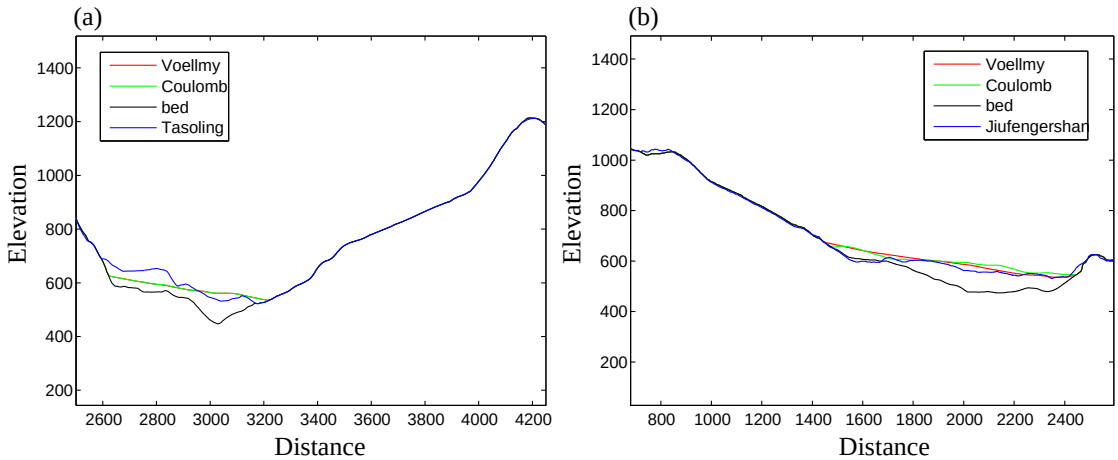


Fig. 3 Final deposits compare to the measurement, on the main profiles. Rheological parameters are taken from Table 1. (a) Tsaoling, (b) Jiufenershan.

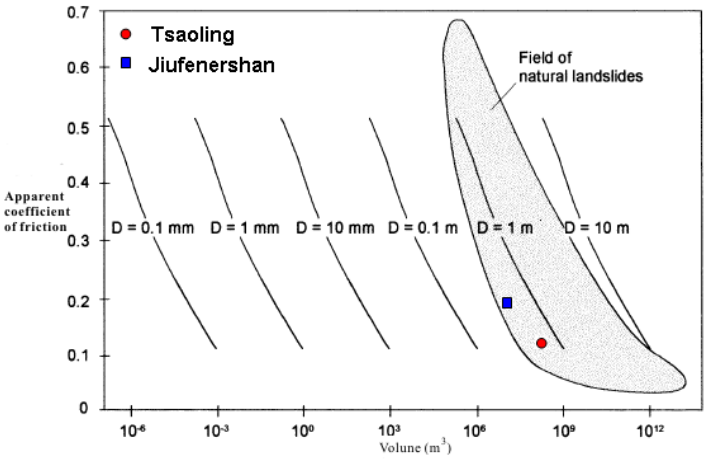


Fig. 4 The apparent friction coefficients of the two landslides compared to the correlation of the friction coefficient versus landslide volume, cf. Legros (2002).

CONCLUSION

We propose a back-analysis method for rheological parameters by minimizing the standard deviation of the deposition height between the simulation and measurement. The rheological parameters describe the basal friction/shear force which is the most important physical factor on the flow run-out distances. Two landslides, Tsaoling and Jiufenershan, triggered by the Chi-Chi earthquake are studied and they both are classified as rock avalanches from the field observation. Neglecting the details of initiation, rupturing processes, the avalanches are assumed thin uniform shallow flows. The scheme is applied with the Coulomb friction law and the Voellmy rheology and the best regressed parameter sets are tabulated in Table 1. For the simple Coulomb friction law, the proposed scheme yields agreeable results, subjected to an acceptable bias, compared with the phenomenal relation of the friction angle and the flow volume. The Voellmy rheology, on the other hand, plays different roles between the two landslides: negligible in Tsaoling but influential in Jiufenershan. There are two reasons suspected to cause the alternation: (1) the Voellmy turbulent coefficient scales with the size of the landslide in contrast to the scaling independent Coulomb friction and (2) the sliding mass of the Jiufenershan landslide contains high water content, hence the turbulent drag manifests itself.

The study verifies that the optimization scheme can be effectively in place of the visual check of the deposit outlines or run-outs. It also provides an objective method for determining more sophisticated rheological models, such as Pouliquen friction, Bingham fluid, Mohr-Coulomb constitutive laws, etc. For applications of the rheology laws in the field, more cases are to be incorporated in future.

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