Experimental Analysis of Roll Waves in Overconcentrated Flow

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Abstract

Water surface instabilities have been observed as wave trains both in laboratory and in field. This phenomenon, called roll waves, is typical of steep and sufficiently long channels (Montuori, 1961; Brock, 1969). Moreover roll waves development concerns not only Newtonian fluid but also mud and debris flow and has been studied most of all in fixed bed condition.

In the present paper the phenomenon of roll waves over mobile bed is presented. A hydraulic flume has been used to study this wave trains in presence of intense bed-load. Measurements of wave features like amplitude, wavelength and celerity have been performed. Finally a comparison with the results known for the fixed bed has been carried out. Particular attention has been dedicated to give physical explanation and interpretation of roll waves development over mobile bed with intense bed-load.

Keywords: roll waves, solid transport, intense bed-load.

Introduction

In steep and sufficiently long channels the uniform flow tends to become unstable and to form waves on the water surface. These waves are called roll waves and are typically characterized by amplitude, wavelength and celerity increasing downward the channel (Brock, 1969; Lamberti and Longo, 2000).

Roll waves development has been observed in spillways, in natural and artificial water courses. Moreover this kind of instability is common not only for the Newtonian fluids, but also for fluids characterized by a different rheological law, such as mud flow and debris flow. In particular, as far as mud flow and debris flow are concerned, the knowledge of the behaviour of this phenomenon is necessary for hazard planning. For example the surface waves observed in debris flow are usually considered using a fixed bed scheme, and a more detailed knowledge of this phenomenon in case of mobile bed can allow a better setting of the problem.

It is possible to distinguish between natural and periodic roll waves. Unlike natural roll wave trains, the second ones are produced by a periodic disturbance of the undisturbed flow. It has been observed by experimental analysis (Brock, 1969) that the evolution of the process is the same in both cases only at a sufficient distance from the beginning of the waves formation. This kind of wave trains can develop both in laminar (*capillary roll waves*) and turbulent flow (*inertial roll waves*) (Balmforth and Mandre, 2004).

Many studies have been developed starting from the pioneer one of Cornish (1910) to other more recent. Usually it is possible to distinguish between analytical and experimental approach. Roll waves are typically characterized by steep front; nevertheless at the beginning their shape is quite smooth because the shock condition is not immediately reached. The growing mechanisms of roll waves over fixed bed are substantially two: a so-called natural one and another one due to the overtaking between waves (Brock, 1961). The first one is related to the natural growth in terms of amplitude along the channel and the second one to the combination of two waves into a single wave with amplitude and celerity greater than the previous ones. This kind of process can be explained observing that waves with different amplitudes are characterized by different celerities too: in particular the bigger the amplitude, the higher is the celerity. Hence different waves can overtake each other.

This process of overtaking can be explained with a momentum balance applied to a shock front moving downstream with constant celerity c over a fixed bed (Fig. 1). It is possible to relate the celerity of shock front c to maximum and minimum water depth (h_{min} and h_{max} respectively) and to the mean velocity of undisturbed flow (U_0).

$$c = U_0 + \sqrt{g \frac{h_{max}}{h_{min}} \frac{h_{max} + h_{min}}{2}} \tag{1}$$

From equation (1), it appears that two waves can have different celerities depending on the value of h_{max} and h_{min}; in particular the one with bigger h_{max} (or smaller h_{min}) has a higher celerity, so it can overtake the other wave (Brock, 1969).

pp. 149–157

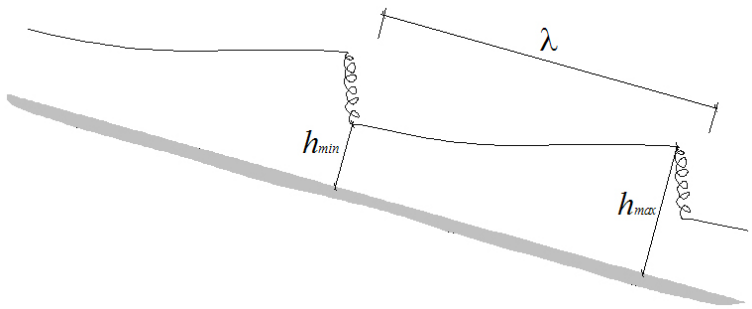

Fig. 1. Scheme of roll waves over fixed bed usually reported in literature.

With regards to roll waves over fixed bed, a stability analysis has often been performed in order to understand which is the condition of formation. Typically the linearized approach for a one-dimensional flow starts from the continuity and the momentum equations that assume the following form for a rectangular cross section:

$$\begin{aligned} \frac{\partial h}{\partial t} + U\frac{\partial h}{\partial x} + h\frac{\partial U}{\partial x} &= 0 \\ \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + g\frac{\partial h}{\partial x} &= g(i_f - i_e) \end{aligned} \tag{2}$$

where U is the mean velocity, h the water depth, i_f the bottom slope and i_e the slope of the energy line.

Perturbing and linearizing the previous equations

$$\begin{aligned} h &= h_0 + h' \\ U &= U_0 + U' \end{aligned} \tag{3}$$

the system (2) reduces to

$$\begin{aligned} \frac{\partial h'}{\partial t} + \frac{\partial h}{\partial t} + U_0\frac{\partial h'}{\partial x} + h_0\frac{\partial U'}{\partial x} &= 0 \\ \frac{\partial U'}{\partial t} + U_0\frac{\partial U'}{\partial x} + g\frac{\partial h'}{\partial x} + gi_f\left(2\frac{U'}{U_0} - \frac{h'}{h_0}\right) &= 0 \end{aligned} \tag{4}$$

From (4) it is possible to obtain the linear instability condition for roll waves over fixed bed in rectangular channels (Dressler, 1949; Iwasa, 1954):

$$Fr_0 \geq 2 \tag{5}$$

where Fr_0 is the Froude number of the undisturbed flow. Relation (5) is obtained considering a value for the exponent of the rate curve, expressed in a monomial form, equal to 3/2.

In addition, the formation and development of roll waves over fixed bed are related not only to condition (5) but also to the channel length (Montuori, 1961; Brock, 1969): in particular the latter has to be sufficiently long in order to allow the growth of wave trains.

Montuori (1961) suggests a formation criterion for roll waves that involves two dimensionless groups:

$$Ve = \frac{U_0}{U_c} \quad e \quad g\frac{Li_f}{U_0^2} \tag{6}$$

The first one is known as Vedernikov number and represents the ratio between the velocity of undisturbed flow U_0 and the critical velocity for the formation of roll waves U_c, while the second one is related to the channel length (L). The instability condition (5) expressed in terms of Ve becomes

$$Ve \geq 1 \tag{7}$$

For rectangular sections and fixed bed Montuori (1961) obtained the following relation:

$$\frac{gLi_f}{U_0^2} = \frac{1 + \frac{2}{3}\frac{1}{Ve}}{1 - Ve}\ln\varepsilon \tag{8}$$

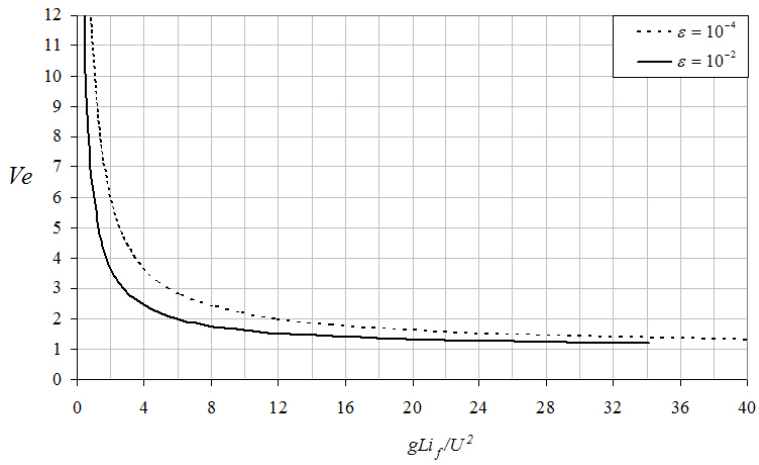

Fig. 2. Criterion for roll waves formation over fixed bed proposed by Montuori (1961). The curves refer to relation (8) calculated with e equal to 10^{-2} and to 10^{-4}.

The parameter ε represents the steepness of the wave: that is the ratio between the inverse of water surface slope near the wave crest and in undisturbed flow condition (Montuori, 1961):

$$\varepsilon = \frac{\left(\frac{\partial x}{\partial h}\right)}{\left(\frac{\partial x}{\partial h}\right)_0} \tag{9}$$

where h is the water depth and x the longitudinal coordinate of the channel. This ratio assumes smaller values as the wave profile becomes steeper. The roll waves development condition suggested by Montuori (1961) is represented in Fig. 2 by the zone above the curves.

While the literature concerning roll waves over fixed bed is relatively rich, any analytical approach to roll waves over mobile bed is absent according to the writers' experience. In the following paragraph a description of this second kind of roll waves will be presented.

Experimental apparatus and measurements

The experiments on mobile bed have been carried out in the Hydraulic Laboratory of University of Trento in a flume 12 m long, and 0.20 m wide. The system allows the recirculation of both water and solid material: a hydrocyclone has been used to separate the solid phase from the liquid one.

The granular material used is characterized by the grain size distribution reported in Fig. 3 (d_{50} = 1.7 mm). Before starting every test, a layer 0.10 m thick of this material has been settled in the flume. The water and the solid discharge are imposed as boundary condition and the system is let to evolve till the achievement of the equilibrium condition, that is when the inlet solid discharge is equal to the outlet solid discharge. In this situation the slope reaches a statistically and stationary value depending just on the liquid and the solid discharge and on grain size.

In a particular range, the equilibrium condition is characterized by roll waves train development along the channel. Those perturbations apart, the bed slope remains almost constant in a statistical sense. In this situation the average water depth, the averaged velocity and the average slope have been measured with traditional instrumentation. In particular the water depth, h_0, and the bed elevation have been measured by level gauges in different positions when the achievement of the equilibrium has been reached. The value of h_0 measured is here considered as the undisturbed water depth.

The measurement of average velocity of undisturbed flow, U_0, has been very delicate. In fact the experimental conditions are quite difficult because they are characterized by low water depth and consistent thickness of bed-load layer (about half water depth). At first conductivity technique has been adopted, but with unsatisfactory results. Therefore the choice has fallen on a particle tracking technique. The main problem involved in this procedure concerns the presence of waves on the water surface that interact with the tracer: the wave passage induces an acceleration to the tracer. Therefore the tracer has to be chosen properly in order to make it flow together with the undisturbed flow. Disks made of two layers of rags with a weight inside have been used (Fig. 4): each disk has a diameter of about 3.5 cm and weighs about 4 g. Using a high speed camera and the tracer mentioned above, the particle tracking has been performed. The passage of roll waves

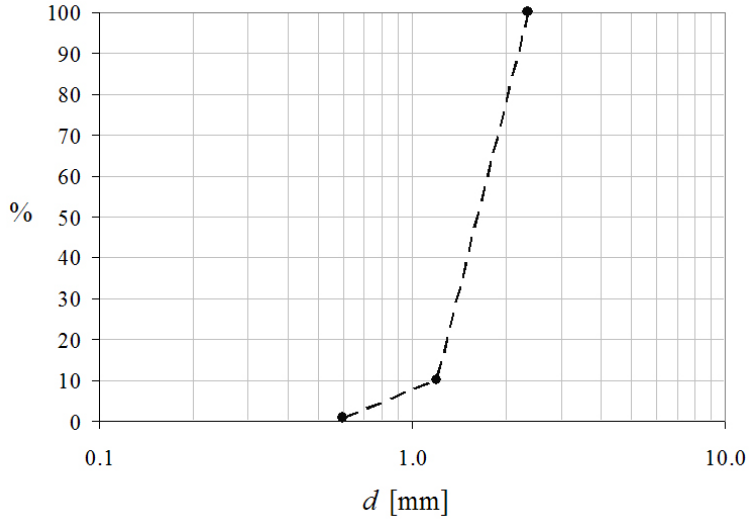

Fig. 3. 3 Grain size distribution of the granular material used.

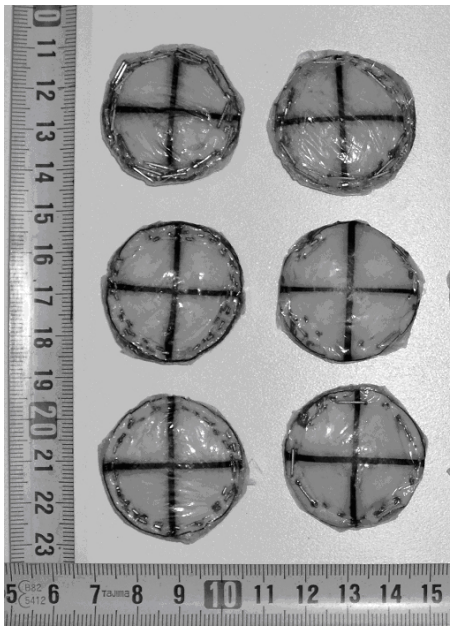

Fig. 4. Tracer used for measuring the flow velocity.

induces an acceleration of the tracer but it is possible to separate these impulses elaborating precisely the data acquired. Knowing the roll waves celerity from experimental measurements, it is possible to filtere the data in order to identify the velocity of basic flow. This procedure has been repeated many times for each test in order to get a representative sample.

The main features of roll waves have been estimated using optical techniques. The evolution of water surface and bed layer have been investigated taking lateral shots with a high speed camera. Then the frames have been analyzed in order to gain information about waves properties, like amplitude, wavelength and celerity.

As already mentioned, the origin of roll waves is related to the hydrodynamic instability of the system. In the present work no external devices have been used to generate roll waves in order to study the so called natural roll waves. Particular attention has been dedicated to the inlet of solid material into the flume; a suitable chute has been used for distributing the material homogeneously along the transversal direction and for avoiding impulsive material inlet, in other words to restrict every possible external disturbance.

All the measurements have been taken once the system had reached the equilibrium condition, when the inlet and outlet solid discharges were equal.

Table 1. Mean features of the tests performed

	Q [l/s]	Q_s [kg/min]	i_f	h_0 [m]	U_0 [m/s]	a_m [m]	λ_m [m]	c_m [m/s]	Fr_0
Test 1	1.13	11.9	0.098	0.010	0.74	0.0071	0.074	1.01	2.42
Test 2	1.43	15.2	0.098	0.011	0.94	0.0078	0.083	1.29	2.91
Test 3	1.86	20.1	0.100	0.013	1.06	0.0068	0.087	1.40	3.02
Test 4	2.50	26.2	0.097	0.014	1.13	0.0063	0.083	1.49	3.05

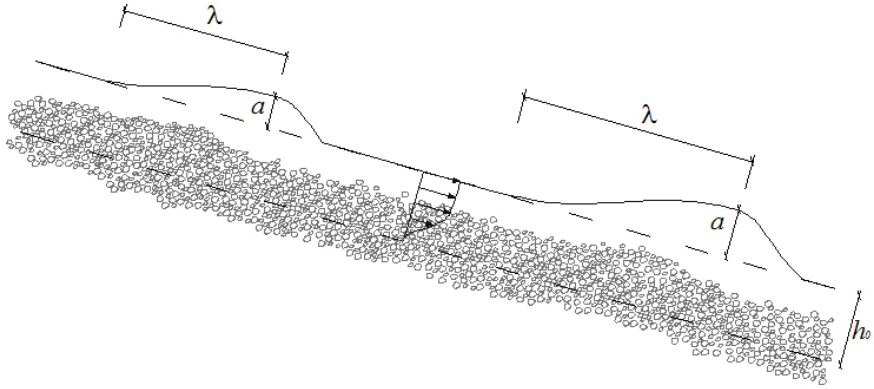

Fig. 5. Scheme of roll waves over mobile bed observed in laboratory.

Experimental test

Some preliminary tests have been done to identify which is the formation range of roll waves. For water discharge lower than 1.0 l/s alternate bars have been observed; as water discharge was increased, roll waves trains have appeared along the channel till the formation of antidunes migrating downstream inhibited their development. Hence due to the presence of bed-forms, roll waves have become less evident.

Once identified the formation zone of roll waves, appropriate tests have been done in order to characterize these wave trains (Table 1). Waves with different amplitude and with different celerities have been noticed and the overtaking process has been observed frequently in the downstream channel zone. This waves identified in laboratory are characterized by a smooth profile and only some of them by a steep front. Brock (1969) used a flume 24 m and 35 m long and observed that waves started to break quite far from upstream; so it is possible that the flume utilized in the present work is not long enough to allow a complete development of roll waves. The waves observed over mobile bed can have a smooth profile also because the system can dissipate energy not only as water surface instability but also as bed modification.

The scheme used to characterize the waves observed in laboratory is shown in Fig. 5. The amplitude a is defined as the distance between the crest and the uniform flow level while the wavelength λ corresponds to the distance between the crest and the point where the wave profile intercepts the undisturbed flow level. The main characteristic of the tests performed can be summarized as shown in Table 1.

The parameters in Table 1 assume the following meaning: water discharge (Q), solid discharge referred to dry material (Q_s), mean bed slope (i_f), water depth of undisturbed flow (h_0), average velocity of undisturbed flow (U_0), average roll waves amplitude (a_m), average roll waves wavelength (λ_m), averaged roll waves celerity (c_m), Froude number of undisturbed flow (Fr_0).

Typically for each trial the relation between dimensionless amplitude and dimensionless wavelength and the one between dimensionless celerity and dimensionless wavelength assumes the trend shown in Fig. 6, which refers to test 2 ($Fr_0 = 2.91$).

It is possible to notice that waves with greater amplitude have a greater wavelength (Fig. 6 on the left). The picture on the right in Fig. 6 shows the increasing of roll waves celerity as amplitude grows. The same behaviour, concerning the relation between celerity and amplitude, was noticed also by Brock (1961) with respect to roll waves over fixed bed.

A comparison of measured roll waves celerity (c), celerity of kinematics (c_0) and gravity waves (c^+) has been performed, as shown in Fig. 7. The celerity of kinematics and gravity waves assumes the following

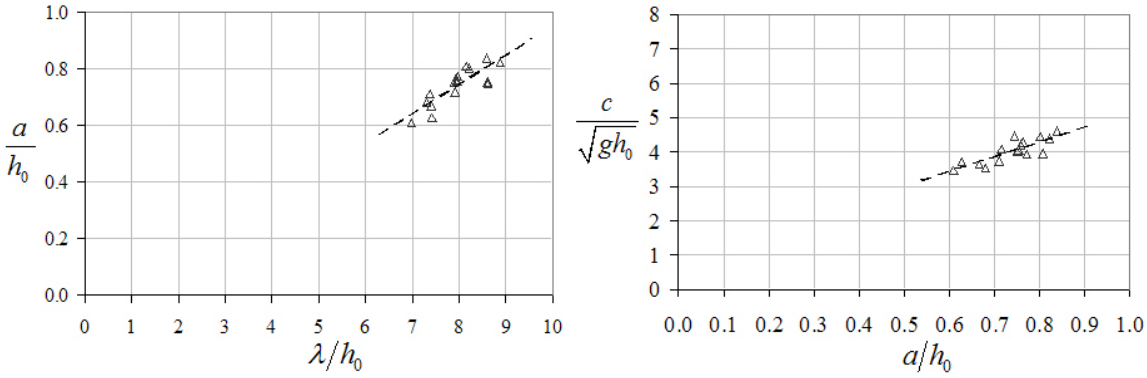

Fig. 6. Test 2 ($Fr_0 = 2.91$): relation between the ratio amplitude (a) over undisturbed water depth (h_0) and ratio between wavelength (λ) and undisturbed water depth (h_0) (picture on the left) and between dimensionless celerity ($c/(gh_0)^{0.5}$) and the ratio amplitude (a) over undisturbed water depth (h_0) (picture on the right).

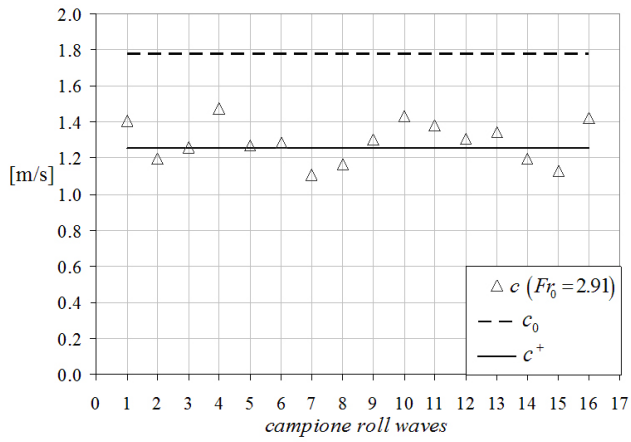

Fig. 7. Comparison between celerity of roll waves observed (c), celerity of kinematics waves (c_0) and celerity of gravity waves (c^+), concerning test 2 ($Fr_0 = 2.91$). The x-axis shows the label of the roll waves identified: for test 2, 16 roll waves have been analyzed.

expression:

$$\begin{aligned} c_0 &= mU_0 \\ c^+ &= U_0 + \sqrt{gh_0} \end{aligned} \tag{10}$$

where m is the exponent of the rate curve expressed in a monomial form. In every test the trend is similar to the one shown in Fig. 7: roll waves celerity observed (c) is very similar to the gravity waves celerity (c^+) that is smaller than the kinematics waves celerity (c_0).

A particular attention has been dedicated to study how the passage of the wave modifies the bed. The frames obtained by lateral shots with the high speed camera have been analyzed: the first layer of motionless grains has been identified and the average bed level has been determined. It was possible to notice the presence of a dig just under the crest of the wave.

Moreover, it seems that there is a sediment transport wave that moves tendentially in delay with respect to the liquid wave. The sediment transport wave has been characterized by its amplitude (a_s) and by the average depth of the bed-load layer ($h_{0,s}$) (Fig. 8). The relation between the ratio sediment transport wave amplitude (a_s) over average bed-load layer depth ($h_{0,s}$) and the same ratio only referred to the liquid wave (a/h_0) is shown in Fig. 9. This data seem not well correlated. The identification of bed-load layer, the definition and the measurements of features concerning solid transport and bed morphology have been very complex due to the difficult experimental conditions; so the remarks made about solid wave dimension and about digging processes have to be considered with caution, as general behaviour of the system.

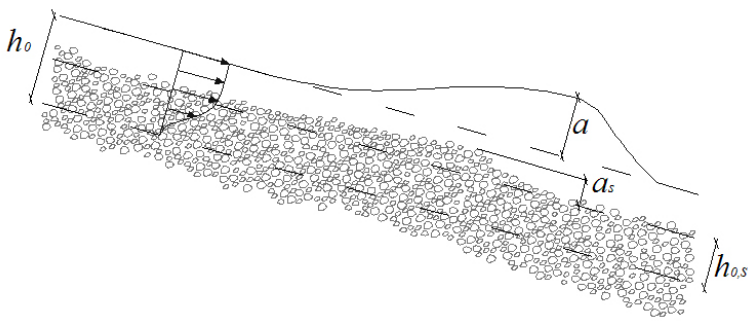

Fig. 8. Characterization of liquid and sediment transport wave.

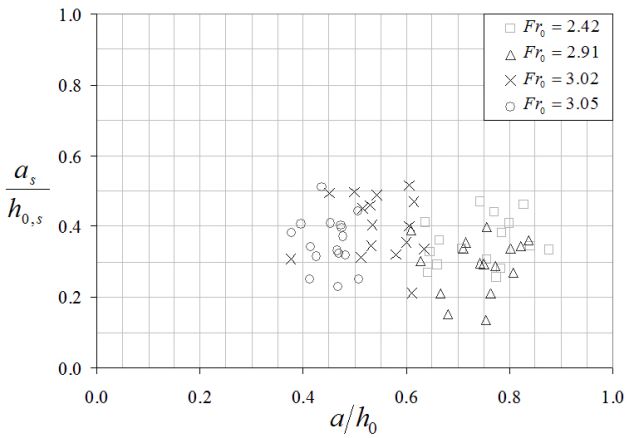

Fig. 9. Characterization of liquid and solid transport waves (picture on the left) and relation between the ratio solid wave amplitude over (a_s) average bed-load layer depth ($h_{0,s}$) and the same ratio only referred to the liquid wave (a/h_0) in the tests performed (picture on the right).

Conclusions

Tests 1 and 2 ($Fr_0 = 2.42$ and $Fr_0 = 2.91$ respectively) are those where roll waves have been observed clearer and more developed. Increasing Froude number (test 3 and 4) roll waves have appeared stretched and less defined. In tests 3 and 4 ($Fr_0 = 3.02$ and $Fr_0 = 3.05$ respectively) bed-forms migrating downstream have been observed and their presence has inhibited the development of roll wave trains.

In Fig. 10 measurements of dimensionless amplitude, dimensionless wavelength and dimensionless celerity for the tests reported in Table 1 are shown. It has been observed that waves with higher amplitude have an higher wavelength. In each test the celerity increases as the amplitude and the Froude number increase (Fig. 10 on the right).

In each trial (Table 1) the development of roll waves has always been observed, but in test 2 the wave trains have appeared clearer and well defined. The ratio between average wavelength (λ_m) and average amplitude (a_m) is considered as wave shape index and the ratio between undisturbed water depth (h_0) and average amplitude (a_m) is considered as wave development index. In Fig. 11 the relationship between these parameters, averaged over each test, versus undisturbed Froude number is shown; for both dimensionless groups it seems that there is a growing trend with Fr_0.

Nevertheless, on the right hand side of Fig. 11 it is possible to notice that for test 3 and 4 the parameter h_0/a_m increases faster because the roll waves amplitude decreases; in fact, in the last two trials roll waves were less developed. It is possible to conclude that roll waves tend to increase their amplitude and their wavelength up to a threshold; beyond this threshold the phenomenon becomes less developed.

The ratio between celerity and velocity of undisturbed flow based on the experimental data results equal to 1.3–1.4 (Table 2). The instability condition (5) is fulfilled in every test done: Fr_0 is significantly grater than 2. The threshold modifies if the value of the exponent m of the rate curve, expressed in a monomial form, changes. Using experimental data a value of m equal to 1.9 is obtained. In this case, according to the linear

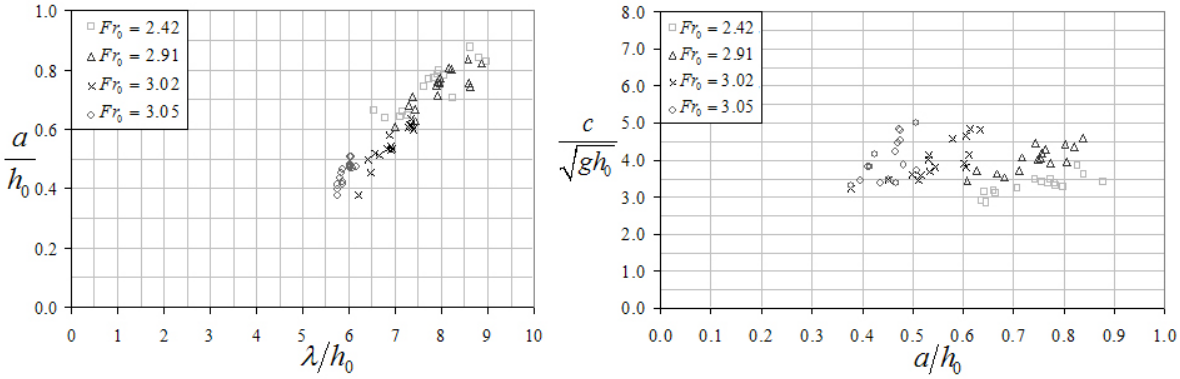

Fig. 10. Relation between the ratio roll waves amplitude (a) over undisturbed water depth (h_0) and the ratio wavelength (λ) over undisturbed water depth (h_0) (picture on the left) and between the dimensionless roll waves celerity ($c/(gh_0)^{0.5}$) and the ratio roll waves amplitude (a) over undisturbed water depth (h_0) (picture on the right).

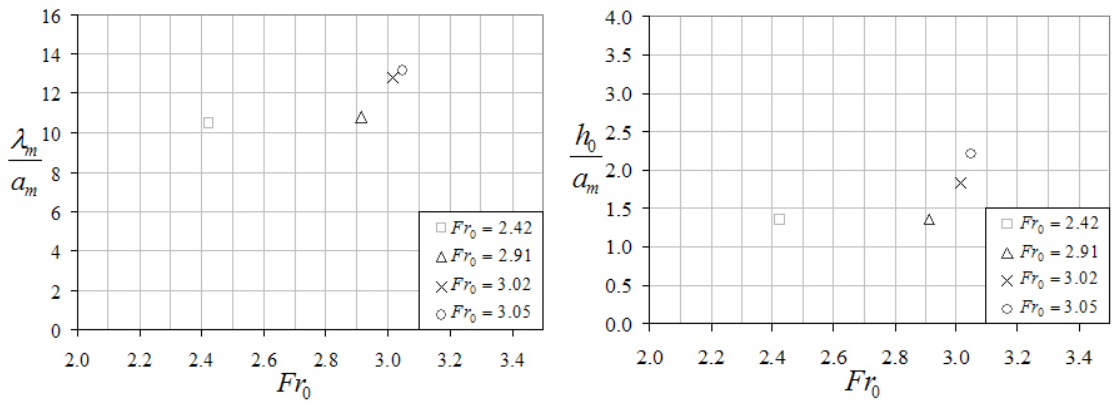

Fig. 11. Ratio between average wavelength (λ_m) and average amplitude (a_m) versus undisturbed Froude number (picture on the left) and ratio between undisturbed water depth (h_0) and average amplitude (a_m) versus undisturbed Froude number (picture on the right).

stability theory, roll waves can develop if Fr_0 is grater than 1.1. It is possible to notice that this condition is satisfied as well.

Using the criterion proposed by Montuori (1961) it has been found out that experimental data fall in the border zone of the curves expressed by relation (8) (Fig. 12). Because this criterion and the relation (5) refer to roll waves over fixed bed, the comparison with data obtained in mobile bed condition has to be considered with caution, that is only as qualitative comparison.

Roll waves observed over mobile bed are characterized by a shape lightly different from those over fixed bed; the second ones have a steep profile while the first ones appear in a smoother form. Nevertheless there are some similarities concerning growing mechanisms: overtaking process and natural growth have been observed in both cases. Particularly interesting might be the study of this phenomena in a longer flume, in order to evaluate a complete development of roll waves.

Finally, the bed-forms observed have influenced the roll waves formation and growth. Therefore the stability criteria for fixed bed should be modified. In particular the analytical approach should be changed probably considering also the Exner equation for the solid phase.

Another attractive aspect concerns the presence of a sediment transport wave, typically in delay with respect to the liquid one, and of a tendency to dig under the wave crest. Nevertheless, the study of bed evolution needs further and more accurate analysis in order to make clearer these phenomena.

This study has been performed using research funds of CUDAM.

Table 2. Ratio between roll waves celerity and averaged velocity of undisturbed flow.

	Fr_0	c/U_0
Test 1	2.42	1.37
Test 2	2.91	1.38
Test 3	3.02	1.32
Test 4	3.05	1.32

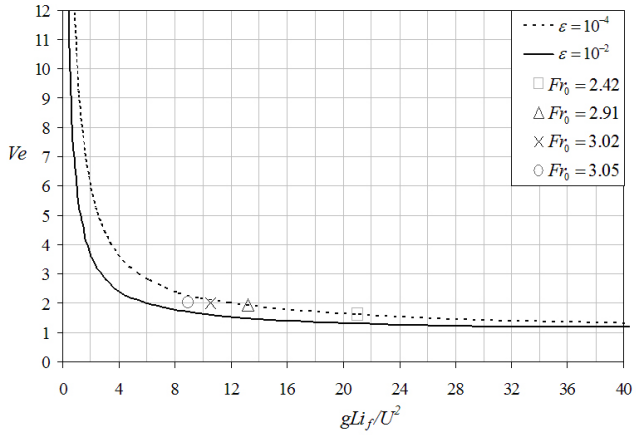

Fig. 12. Comparison with experimental data referred to mobile bed and the Criterion proposed by Montuori (1961) for roll waves over fixed bed.

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