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SNOW MOTIONS

INTERACTION BETWEEN A SNOWPACK AND A SNOW NET SYSTEM

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ABSTRACT

The present study deals with snowpack motions and its interaction with a flexible supporting structure. The final aim of this research is to evaluate forces induced into a snow net system by its interaction with a snowpack. This calculation will allow to obtain a better engineering design of these structures. First, a 2D continuous coupling model was developed. This description is based on the coupling of a finite element code and a continuous representation of the structure (a cable in 2D). The FEM code, in which the downstream boundary conditions correspond to the cable representation, describes the snowpack motions. A Coulomb friction law is used as snowpack – cable interface relation. A parameter analysis was performed to investigate the influence of the friction angle, the cable inclination (with respect to the slope) and gravity components. The forces at each node increase with increase of the friction. After a transient phase, the snow mesh and the forces components tend to an asymptotic state at the same time, combined to a low settlement.

Key words: snowpack, snow creep, snow net system, interaction, finite element method

INTRODUCTION

Protection systems against avalanches are divided into two groups. One, called passive, controls the avalanche flow by its velocity, height or length of its run out distance. The second, called active protection, stabilises the snowpack to avoid triggering of the avalanches as snow rakes or snow net systems (Fig.1).

Fig1: Snow net system

Various studies have investigated snowpack interaction with obstacles, which are often rigid structures such as mast (Larsen, 2000; McClung,

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1984; Margreth, 1990 and 1995). When the structures are flexible, such as snow net systems, their final shapes are commonly a priori assumed (Margreth, 1990). However, the adopted shapes, which vary according to the snowpack conditions, may be quite different from the ones observed in the field (Nicot et al, 2002 and 2003). Also, prescribing a fixed geometry for a deformed net can possibly infer wrong results as regards to the forces exerted by the net on the anchors, leading then to a poor design. A more detailed study of the snowpack – net interaction is thus required.

As a 3D continuous model is a time consuming approach, first, a 2D model has been developed. This description is a coupling between a 2D finite element code for the snowpack and a continuous representation for the net reduced to a cable. The latter is based on an incremental method and used as downstream boundary conditions for the snow mesh. The interaction between the snowpack and the wire is founded on a Coulomb-type friction law.
In this coupling, different parameters such as friction angle or cable inclination are used. To determine their influences, a parameter analysis was performed. Moreover, the full gravity is taken into account in the first computations, even occurring damage are due to the snowpack motions parallel to ground.

THE 2D SIMULATION

The 2D model is divided into two parts. The snowpack deformation is simulated by a finite element code, using a continuous constitutive relation for the snow creep. The snow net is reduced to a simple wire.
The interaction between the snowpack and the cable results in forces applied at the snowpack-cable interface. These forces are generated by the cable elastic deformation and with interface condition. They are calculated by the Coulomb-type friction law. This law models the gliding, which is likely to occur between the snow and the cable.

THE SNOW

The finite element code used for the snowpack allows computing the velocity field in the snowpack for a given field of snow density. It uses 6-nodes triangular elements with a quadratic interpolation of the velocities and a linear interpolation of the pressure. The implemented viscoplastic constitutive relation, based on metal powder sintering (Duva and Crow, 1994) was proposed by Gagliardini and Meyssonnier (1997).
The expression of the stresses is as follows (Eq.1):

$$\sigma_{ij} = B^{-1/n} \dot{\varepsilon}_D^{(1-n)/n} \left(\frac{2}{a} \dot{e}_{ij} + \frac{1}{b} \dot{\varepsilon}_{kk} \delta_{ij} \right) \tag{1}$$

$$\dot{\varepsilon}_D = \frac{2}{a} \dot{e}_{ij} \dot{e}_{ij} + \frac{1}{b} \dot{\varepsilon}_{kk} \tag{2}$$

In which $\dot{\varepsilon}_D$ is the second invariant of the strain-rate tensor $\dot{\varepsilon}$ defined as:
where $\dot{e}_{ij}$ are the deviatoric stain rates defined by Eq.3:
The other terms: n, B, a and b, are the parameters. n is a positive integer whom the general value is equal to 3. B, the fluidity, inverse of the viscosity, is a temperature dependant

$$\dot{e}_{ij} = \dot{\varepsilon}_{ij} - \frac{1}{3} \dot{\varepsilon}_{kk} \delta_{ij} \tag{3}$$

parameter and its value is commonly equal to 20 $MPa^{-3}a^{-1}$ (Lliboutry and Duval, 1985). *a* and *b* are conveniently expressed as functions of the relative density of snow defined as the ratio of the actual density of snow and that of the pure ice (917 $Kg.m^{-3}$). The expression of *a(D)*

$$\begin{cases} a = \dfrac{\left(1 + 2\left(\dfrac{1-D}{3}\right)\right)}{D^{2n/(n+1)}} \\ b = \dfrac{3}{4}\left[\dfrac{(1-D)^{1/n}}{n\left(1-\left(1-D\right)^{1/n}\right)}\right]^{2n/(n+1)} \end{cases} \qquad \text{for } D > 0{,}81 \qquad (4\text{-a})$$

and *b(D)* were taken as follows:

The expressions for higher densities derive from Duva and Crow's theoretical work (1994). For lower densities, *a(D)* and *b(D)* were fitted with respect to Landauer's experimental results (1957) by Gagliardini and Meyssonnier (1997).

Figure 2 shows the finite element mesh used to model the snowpack lying between two rows of snow nets. The boundary conditions are as follows:

- The upstream boundary and the upper surface of the snowpack are considered as free surfaces: i.e. no stresses or velocities are prescribed at the corresponding nodes,
- The base of the snowpack is assumed to glide freely on the ground,

$$\begin{cases} a = \exp\left(-19.25D + 16.03\right) \\ b = \exp\left(-22.51D + 16.75\right) \end{cases} \qquad \text{for } D < 0{,}81 \qquad (4\text{-b})$$

- The downstream boundary nodes are submitted to external forces (the snowpack – cable interaction forces),
- Creep of snow occurs owing to gravity.

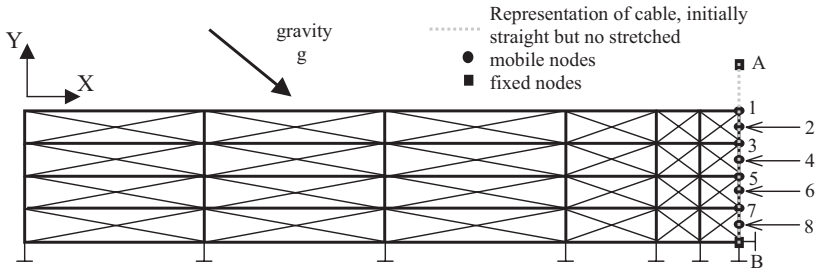

Fig2: Mesh for the snowpack and the cable used in the computations
Numbered downstream nodes of the snowpack correspond to interface nodes of the cable

THE INTERACTION

At each step of the iterative process, with the time step τ, the finite element code gives us the velocity field of the snowpack, and then the snowpack deformation. The downstream of the latter gives us directly the new shape of the cable, as the downstream boundary nodes of the snow mesh correspond exactly to the nodes of the cable (called interface nodes). The elastic

response of the wire generates interaction forces at the snowpack – cable interface nodes, which then react on the snowpack motions.

The coupling of the cable and snowpack models is thus performed as follows.

- At time t, the snowpack model has run for a given set of interaction forces applied at its downstream boundary nodes.
- The resulting velocity field allows to compute the new shape of the snowpack at time $t+\tau$, and, in particular, the displacement of the downstream nodes during τ.
- These displacements are prescribed to the cable nodes, and the model for the cable (presented in the next section) provides the interaction forces at $t+\tau$.
- These forces are taken as new downstream boundary conditions for the snowpack at $t+\tau$, and the whole process is repeated.

The initial condition for snowpack – cable coupling is that the cable joins the two fixed points A and B (Figs 2 and 3) as a straight line and is not stretched. Thus, the process is started by the snowpack velocities computation assuming a free downstream boundary.
It is also assumed that there is no penetration of the cable into the snowpack, and that a Coulomb-type friction law applies at the snow – cable interface. The cable nodes correspond exactly to the downstream snow mesh nodes.

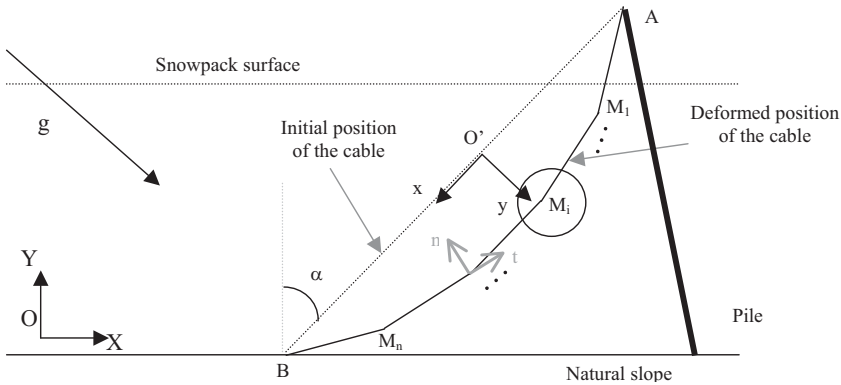

Fig.3: Schematic of the 2D snowpack – cable interaction.
The global reference frame {X, Y} is with its X-axis along the slope.
The local reference frame {x, y} is attached to the cable initial position.
The frame linked to the studied node (n, t), in grey

THE CABLE

The basics of the cable model may be explained first by considering the following problem. Let us consider a cable stretched between two fixed points O and A (Fig.4). If a displacement is applied at a fixed abscissa of the cable, when the displacement is important enough, the cable could glide at the applied displacement point (along the MM' section, in Fig.4).

As a consequence, the cable sections are now OM' and M'A. In this case, as we know the localisation of the points, we can be geometrically calculated the final lengths of the sections. But, the initial ones, which were initially x_M and L-x_M, have evolved to $x_{M'}$ and L-$x_{M'}$. These

initial lengths are unknown as they depend on friction angle or occurred gliding. Thus, the problem in this principle is to calculate the initial lengths of the cable parts.

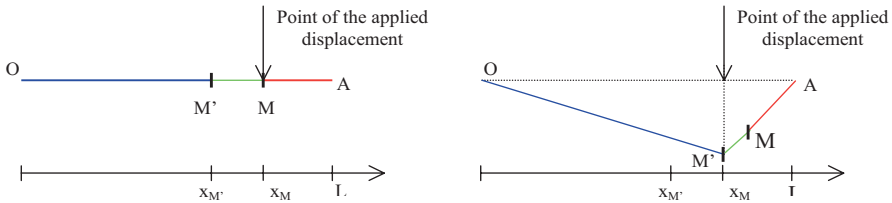

Fig.4: Schematic of the basic principle of the cable modelling.
Left: Initial position. *Right*: gliding occurs and, as a consequence, tensile forces evolve as function of initial length evolution (unknown). The final lengths are geometrically calculated.

For the interaction with the snow mesh, the cable is described by N mobile interface nodes (plus two more fixed nodes to simulate the anchor and the top of the pile, Figs 2 and 3), which move to adapt the shape of the cable to that of the downstream part of the snowpack. The nodes of the cable exactly correspond with the downstream snow mesh nodes at each time *t*.

The cable representation is based on an incremental method. We assumed that the incremental displacements of the nodes at each time step are small enough to consider that each tensile force T(t+τ) at time $t+\tau$ can be regarded as the tensile forces T(t) at time *t* plus an incremental part ΔT(t+τ), as follows:

$$T(t+\tau) = T(t) + \Delta T(t+\tau) \qquad \text{(Eq.5)}$$

As all tensile forces are basically defined by equation 6:

$$T = ES\frac{l_f - l_o}{l_o} \qquad \text{(Eq.6)}$$

where *E* is the steel Young modulus, *S* is the surface of the wire, l_f is the final length of the studied cable section and l_o the initial one. Then, the incremental tensile force is derived from Eqs.5 and 6:

$$\Delta T(t+\tau) = ES\left(\frac{l_f(t+\tau) - l_o(t+\tau)}{l_o(t+\tau)} - \frac{l_f(t) - l_o(t)}{l_o(t)}\right) \qquad \text{(Eq.7)}$$

After using a Taylor development to determine a relation between the initial lengths at time t and t+τ and its increment one, dl_o:

$$\frac{1}{l_o(t+\tau)} = \frac{1}{l_o(t)\left(1 + \frac{dl_o}{l_o(t)} + \cdots\right)} \approx \frac{1}{l_o(t)}\left(1 - \frac{dl_o}{l_o(t)}\right) \qquad \text{(Eq.8)}$$

$$\Delta T(t+\tau) = ES\left(\frac{l_f(t+\tau) - l_f(t)}{l_o(t)} - dl_o(t+\tau)\frac{l_f(t+\tau)}{l_o(t)^2}\right)$$

Equation 7 becomes:
And equation 5, for the node M^i:

(Eq.9)

$$T^i(t+\tau) = T^i(t) + \frac{ES}{l_o^i(t)}\left(l_f^i(t+\tau) - l_f^i(t) - dl_o^i(t+\tau)\frac{l_f^i(t+\tau)}{l_o^i(t)}\right) \qquad \text{(Eq.10)}$$

The development of these equations changes the unknowns. Now, we are looking for the incremental lengths dl^i_o of each cable parts between M^i and M^{i+1} at each time step.

To resolve the equation for each segment, we have first to take into account the gliding condition. This condition is founded on an inequality of force components (R_n and R_t, the normal and tangential force components) on the frame linked to the studied node (Fig.3). These components are geometrically calculated from the tensile forces (Eq.10) of each cable part for the nodes M^i as follows:

$$\begin{cases} R_n^i = \left(T^i + T^{i-1}\right)\sin\left(\dfrac{\theta^{i-1} - \theta^i}{2}\right) \\ R_t^i = \left(T^i - T^{i-1}\right)\cos\left(\dfrac{\theta^{i-1} - \theta^i}{2}\right) \end{cases} \qquad \text{(Eq.11)}$$

where θ^i is the angle of segment M^i - M^{i+1} relative to the x-axis of the local reference frame attached to the initial position of the cable (Fig.3).

If no gliding occurs, the friction condition corresponds to Eq.12, with φ, the friction angle:

$$R_t < R_n \tan\varphi \qquad \text{(Eq.12)}$$

and the initial length is still the same between times t and t+τ. If gliding occurs, the previous friction condition becomes:

$$R_t = R_n \tan\varphi \qquad \text{(Eq.13)}$$

and the initial lengths are changed.

In the case of the interaction between the cable and the snowpack (Fig.3), N interface nodes are in interaction with the snow mesh. Let us consider a part of the cable with Ng gliding nodes delimited by two non-gliding nodes. The number of unknowns is Ng+1 (number of cable sections), whereas, from Eq.12, we obtain one equation for each gliding node, thus Ng equations. Then, one equation misses. It is obtained by writing that the sum of the initial lengths of segments lying between the two non-gliding nodes does not vary during τ, i.e. the total increment for the initial lengths is still the same (Eq.14):

$$\sum_{i=k_0}^{k_o+Ng} dl_o^i\left(t + \tau\right) = 0 \qquad \text{(Eq.14)}$$

The description is coupled with the FEM code and the previous boundary conditions. Then, a parameter analysis was performed on the friction angle.

APPLICATIONS

The snowpack – cable model has been used to determine the influence of three parameters, which can influence the behaviour of the coupling. The friction angle is a parameter used to model the interaction between the cable and the snow. The angle between the cable and the normal to the slope is defined from different papers (Margreth, 1990; Norme Française, 1992) to be between 25° and 35°, how this angle influences the coupling. Moreover, from experiments, damage occur mainly due to the snowpack motions parallel to the ground, we should see that the normal to the slope movements of the snowpack can be neglected to the parallel to the ground ones.

The main purpose of this simulation is to obtain the interaction between a snowpack and a cable for a given time. We do not try to simulate their interaction during all the winter, just for different instants during winter.

Fig.5: *Left*: Monitored snow net system in Flaine (74, France)
Right: Monitored anchor of the net system

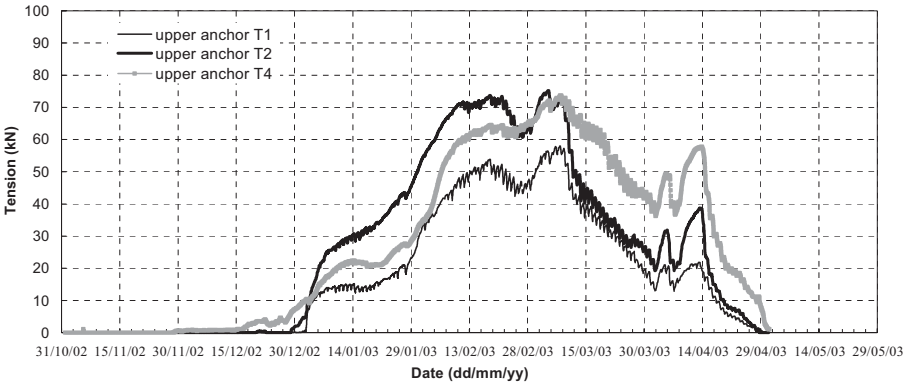

Fig.6: Sensor tensions at Flaine, Winter 2002/2003

The friction angle range is limited between 15° and 45° to a realistic snowpack - cable interaction. In in-situ measurements done at Flaine (74, France, Fig.5), we obtain the forces at anchors as in figures 6. When we look closer to these measurements and couple them with temperature ones, we can see the effect of the day variation of the temperature in the forces (Fig.7). Lower is the temperature, higher are the forces. When the snowpack is likely to stick to the cable, the forces are more important. Thus, in the simulations, we should avoid the case with a null friction angle, and limit φ to 45°, as snow could also slide along the cable even if the snow is frozen.

In the first computations, a simple mesh (Fig.2) is used for a homogeneous snowpack of 20m long and 4m high with a relative density equals to 0,4. The full gravity is taken into account. The cable is first assumed to be perpendicular to the slope.

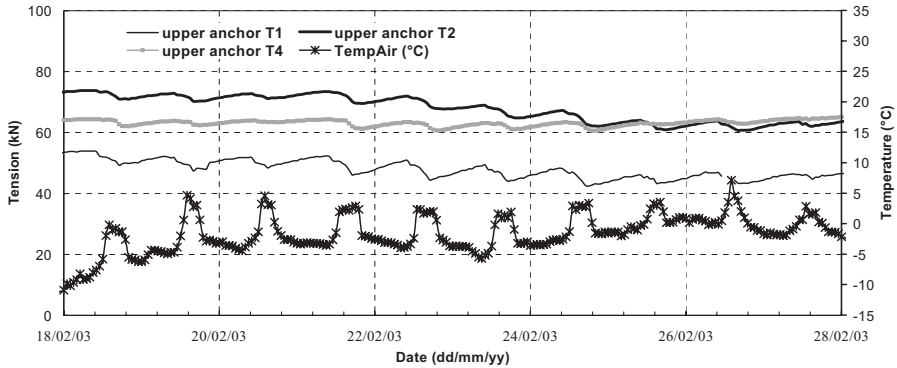

Fig.7: Detail of figure 6 from February the 18th to the 28th, coupled the air temperature

RESULTS

The previous snow mesh (Fig.2) was used in these first computations. We focused first on three different friction angles (15°, 30° and 45°). On figures 8 to 11, the force components are calculated in the X-axis and Y-axis (from the frame along the slope, Fig.3) for the nodes 3 and 8 (Fig.2).

The main aspects of the graphs are the oscillations and the asymptotic state. The oscillations are due to the chosen model for the interaction. The Coulomb-type friction law induces an exchange of forces between nodes when the tangential components are bigger than the normal ones for the studied nodes (Eqs.12 and 13). These oscillations correspond to a transient phase. These oscillations do not simulate any observed behaviour of the interaction, it is due to the chosen interaction model. Then, the obtained asymptotic state corresponds to a stabilisation state for forces and also for the cable deformation (Figs.12 – 14).

The X and Y force components decrease when the friction angle, φ, increases (Figs.8-11). As they are mainly negative, in absolute value, they increase. The friction angle increase the forces applied to the cable.

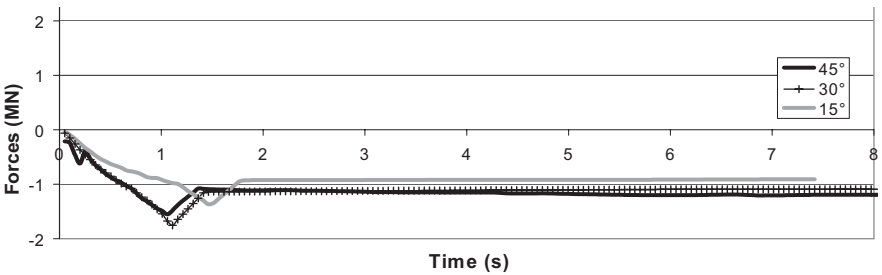

Fig.8: Force component along the X-axis for the node 3 for the friction angles of 45°, 30° and 15°

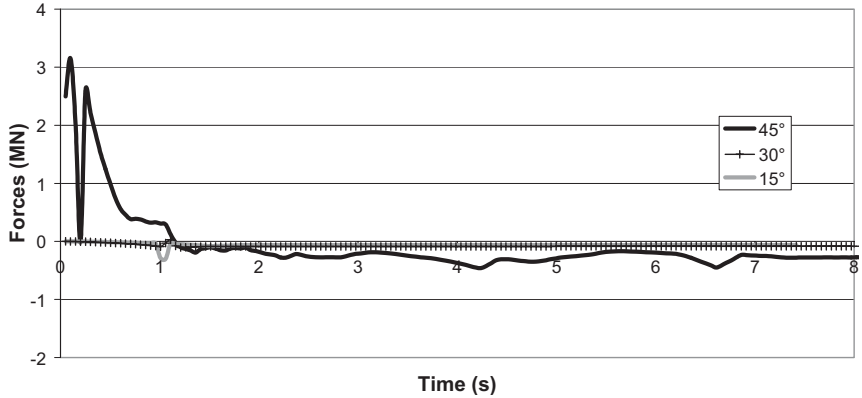

Fig.9: Force component along the Y-axis for the node 3 for the friction angles of 45°, 30° and 15°

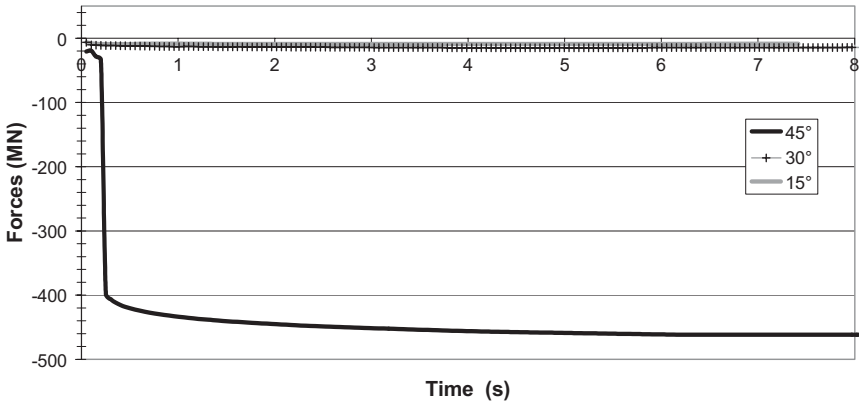

Fig.10: Force component along the X-axis for the node 8 for the friction angle of 45°, 30° and 15°

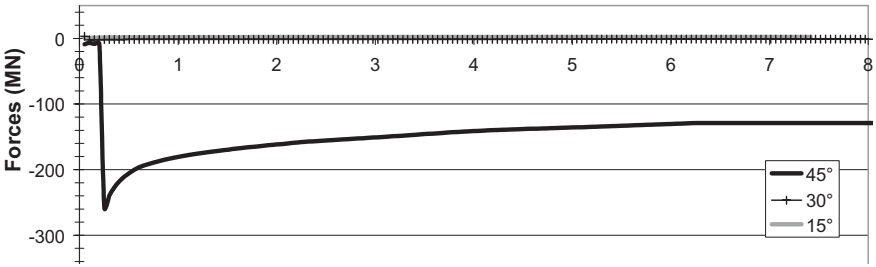

Fig.11: Force component along the Y-axis for the node 8 for the friction angles of 45°, 30° and 15°

In figures 8 and 9, representing the force components for the node 3, and in figures 10 and 11, representing the force components for the node 8, the range for the forces cannot be the same as the force component on node 8 begin to be very important for φ equals 45°. We can easily

imagine when the friction angle is higher than 45°, the force components on the node 8 will be higher.

The friction angle decreases the force components of the nodes but less for the upper nodes than the others (the absolute value increasing). The force becomes to be too high when φ equals 45° and over.

About the snow mesh deformation of the snowpack, it is described by figures 12 to 14. The snow densification occurs slowly. It does not have enough time (only 8 seconds in the computations) to occur, then, we can assume that the densification of the snowpack has no effect in the interaction.
In this short time, we can also observe the normal to the slope settlement can be neglected in respect with the other part. This shows this settlement component has no noticeable effect on the interaction in this configuration.
In the downstream part of the mesh, where the cable interacts with the snow, we can see a stabilisation of the position of the snow elements (which represent also the interface nodes). The downstream tends rapidly to an asymptotic state. After that, only densification occurs.

The friction angle influences softly the snow mesh deformation. The upstream is quasi unchanged, but for the interface one, the upper nodes go up when the friction angle increases. Also, at the same time, the lower nodes tend to go down. Finally, the downstream part of the snow mesh blows up with the increase of the friction.

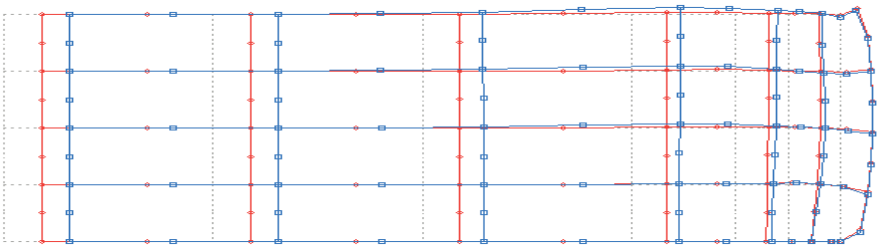

Fig.12: Snow mesh deformation with a friction angle of 45° and the full gravity

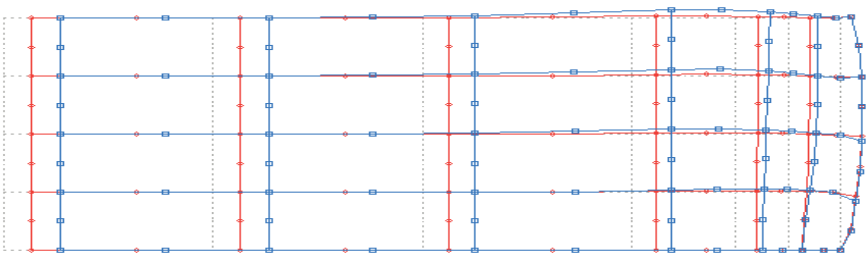

Fig.13: Snow mesh deformation with a friction angle of 30° and the full gravity

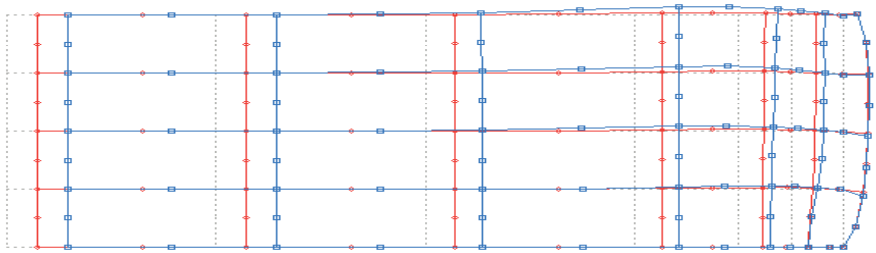

Fig.14: Snow mesh deformation with a friction angle of 15° and the full gravity

CONCLUSION

A 2D model for the interaction between a snowpack and a flexible structure was presented. This model is based on a FEM code, in which a viscoplastic constitutive relation was implemented, and a continuous description of the net system reduced to a simple wire. The description of the cable uses an incremental method where each incremental displacement of the node induces an increment of the tensile force for the corresponding cable section. The interaction between the snow and the cable was simulated by a Coulomb-type friction law.

The model uses different parameters, which the influence in the computation has to be determined: the friction angle, the inclination of the cable and the effect of the normal to the slope snowpack motions. In a first approach, a simple snow mesh was used. Only, the influence of the friction was presented, and some information has been obtained from these first results about the effects of the effect of gravity components on the interaction.

The increase of φ increases the force components for each interface node, in absolute value. After a transient phase, where some oscillations occur due to the chosen interaction, an asymptotic state is reached.

An asymptotic state is also reached, at the same time, by the snow mesh deformation. Densification of the snow takes place softly during the calculations (in 8 seconds). The normal to the slope settlement does not have any significantly effect in the snowpack – cable interaction, for this configuration.

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