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ROLE OF TREE ROOTS SYSTEM FOR SLOPE FAILURE OBTAINED FROM TWO DIMENSIONAL ANALYSIS

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ABSTRACT

The objective of this paper is to disclose the role of the tree roots system in slope stability using analytical examinations. Variations of the diameter and the area ratio of roots depending on the position of root within the soil were determined using two actual Japanese cedar trees(*Cryptomeria japonica* D. Don). Distributions of the diameter and the area ratio of roots were assumed to be a hemisphere shape centered at the center of the stump. The role of this system was clarified by means of the Spencer method for two dimensional analysis. The conventional studies by many researchers may adopt fixed and linear slip surfaces in stability analysis. In this study, the safety factor for slope failure was optimized to determine the critical slip surface considering the non-circular as well as arbitrary slip surface. The analytical examination using model slopes, which consist of homogeneous and isotropic soil material, revealed that a tree located at the toe of a slope is effective by far in the stability slope, and the role of this system is the function of increasing the critical safety factor for slope failure, although the depth of slip surface (i.e. a scale of slope failure) increases.

Key words: tree roots system, slope stability, critical slip surface, two dimensional analysis, Spencer method

INTRODUCTION

Forest is concerned with the occurrence of sheet erosion and shallow slope failure. It is well known that the presence of floor plants and the high permeability of forest soil are effective so far to restrain or prevent sheet erosion. However, degree of influence on mass movement (i.e. slope failure) of a tree roots system, being a major component of a forest, cannot be disclosed

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sufficiently. In the situation that forest area is disappearing gradually on a global scale, the evaluation of this influence on land environments, such as the occurrence of sheet erosion and shallow slope failure, becomes extremely important.

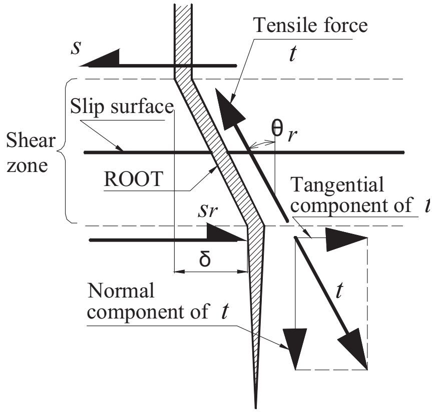


Fig.1: Reinforcement mechanism by a root

Influences of the tree roots system on slope failure may include binding force to a soil, root reinforcement in shear stress, soil moisture modification, windthrowing of a tree, root wedging into rock mass, etc.(D. H. Gray *et al*., 1982). These influences, from the point of view of restraining slope failure, are classified into positive factors and negative factors. In this study, the positive factor that the tree roots system restrains slope failure by the action of the root reinforcing the shear stress will be examined, and the role of the tree roots system in slope failure will be clarified.

Studies on the action of a tree roots system in restraining slope failure have been conducted by several researchers (e.g. Tsukamoto, 1986). In most of these studies, the slip surface formed by slope failure is assumed to be a straight line and the location is assumed to be a line along the boundary between the upper weathered layer and lower basement stratum. However, in slopes with trees which do not have obvious slip surfaces under the geological conditions or which are formed by an embankment, conventional methods cannot evaluate the stability of slope failure, hence the role of the tree roots system in the stability of such slopes is not well known. Namely, for such a slope, the determination of the slip surface should be analyzed on the assumption that the slip surface is not only a straight line but also an arbitrary line, and a change in the slip surface depending on the soil strength and the distribution of the tree roots system should also be analyzed. Conventional methods cannot deal with these points in their analysis.

Accordingly, in this study, by means of a method including the Spencer method for two dimensional section for a non-circular slip surface, which can determine a critical slip surface, the role of a tree roots system on slope failure will be examined. Distribution of diameter and area of roots in the analyses are expressed as quantity models which are defined based on actual removed tree roots. These models can estimate the root reinforcement in shear force due to the tree roots system in soil.

MODEL OF ROOT REINFORCEMENT

A root in a soil mass reinforces the soil in terms of failure in shear (hereafter will be called 'root shear resistance force(or stress)') by a tensile force of a root caused by relative shear displacement of soil masses. In this paper, this mechanism, which has been utilized by many researchers (e.g. Wu *et al*.,1979) is adopted to evaluate the effect of a root to reinforce the shear resistance of soil. Fig.1 shows that a root perpendicular to the shear plane deforms by relative displacement (δ) between the upper soil mass and lower soil mass. If a root is inclined at an angle (θr) to the vertical line with appearance of a shear zone along the slip surface, whose thickness depends on the stiffness of a root and soil masses, the tensile force (t) in a root occurs. A root shear resistance force (sr) can be obtained from the sum of the component of the t tangential to the slip surface and the shear resistance caused by the action of the internal friction angle (φ) under the component of the t normal to the slip surface. A root shear resistance force can be expressed based on this mechanism as follows:

$$sr = t \cdot k \qquad (1)$$

$$k = (\sin\theta r + \cos\theta r \cdot \tan\varphi\ )$$

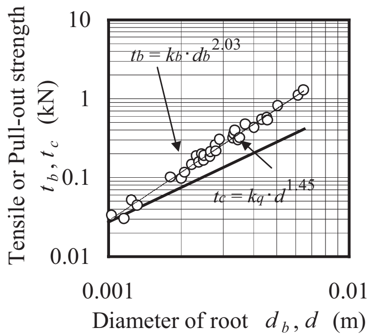


Fig.2: Relationship between tensile or pull-out strength and diameter of a root

Thus, if the internal friction angle (φ) is given, a root shear resistance force (sr) can be determined by use of the tensile force (t) and the inclination angle (θr) of a root.

Method of determination of the tensile force(t) which is an unknown value in Equation(1), will be examined. Hayashi (1998) investigated the relationship between the tensile strength (tb) and the diameter (db) of a root as shown in Fig.2, and derived the approximate equation shown as a straight line in Fig.2, written as:

$$tb = kb \cdot db^{2.03} \qquad (2)$$

where kb is a coefficient defined by the tree species, and its value is $3.38 \cdot 10^4$ in the case of the Japanese cedar expressed by the dimension unit of kN and m.

On the other hand, Tsukamoto(1986) examined the relationship between the diameter (d)of a root at the point of pulling out and the pull-out strength (tc) of a root by using Japanese cedars aged 30 and 20 picked from the Experimental Forest, Tokyo University of Agriculture and Technology and derived the approximate equation shown as the bold straight line in Fig.2, written as:

$$tc = kq \cdot d^{1.45} \qquad (3)$$

where kq is a coefficient defined by the tree species, and its value is $6.14 \cdot 10^2$ expressed by the dimension unit of kN and m.

Besides the above relationship, when the adhesion force on the surface between a root and soil is smaller than the tensile strength of a root, a root must pull out. However according to the author's empirical study, such phenomenon may occur only in a situation of pulling out of the rootlet in the root tip, hence, would be not general.

Comparing Equation(2) with Equation(3), the tensile strength (tb) is different from the pull-out strength (tc) of a root in spite of same diameter (db , d). As the exponent of Equation(2) is 2.03 (i.e. around 2.0), the tensile strength (tb) is in proportion to the area of a root whereas the exponent of Equation(3) is 1.45, the pull-out strength increases at lesser rate for the increasing rate of the area of a root. This means that a breaking point of a root may occur at a point of smaller diameter within the soil mass than the diameter (d)at a point of pulling. Because if the tensile force exceeds the skin frictional strength along the adhesion surface between root and soil, a part of the adhesion surface of the root slips. In actual cases that tensile force mobilizes into a root within the soil mass, a root may display the pull-out strength (tc) given by Equation(3) rather than the tb except for a root penetrating into a crack, etc. of a rock mass.

Next, it is difficult to determine a unique angle of θr given by Equation(1) caused by the shear displacement of soil masses, because the angle of θr depends on an angle between the extending direction of a root and the direction parallel to the shear plane and on the stiffness of a root and the soil mass. Wu *et al*. (1979) proposed the angle of θr as 48deg.-72deg. based on the experimental investigation using the test of tensile deformation of roots. By using these values in given soil conditions, the value (θr) of the parenthesis included in Equation(1) can be estimated, and the root shear resistance force (sr) can also be evaluated.

The root shear resistance stress (cr) on the shear plane can be written by:

$$cr = \sum_{i=1}^{n} sri \tag{4}$$

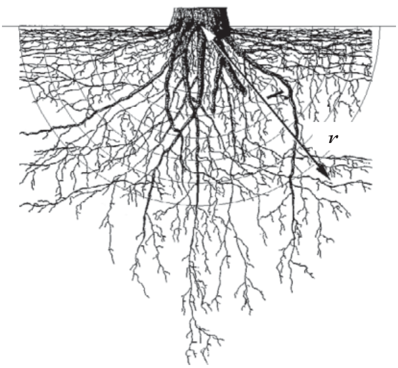
where n is the number of roots per unit area. The effect of the root to reinforce the shear resistance of soil in general is estimated as the term of cohesion in Coulomb's failure criterion. According to this concept, the shear strength (τ) of soil can be written by:

$$\tau = c_r + c + \sigma \cdot \tan\varphi \tag{5}$$

MODELS FOR ROOT DISTRIBUTIONS

To determine sr in Equations (1) and (4), several quantities with respect to the tree roots system at arbitrary points within the slope mass should be known (i.e. the diameters, the number of roots and these distributions). This chapter will deal with the modeling for the quantity and distribution with respect to the tree roots system.

Situation of the distribution of the roots within the slope mass is extremely complex because of effect of various factors such as the species, age, growing environment (i.e. climate, geological condition, nature of soil and moisture of soil). To reveal such distribution of a tree roots system, the measurement needs to be performed by using actual excavated trees from various conditions. In this paper, 35-year-old Japanese cedars are utilized.



(Illustration is after Karizumi(1987))

Fig.3: Assumption of distribution of tree root

If several quantities of tree roots systems can be quantified, the analyses may become convenient in the estimation of root reinforcement for slope failure. Then the variation of distribution of several quantities of tree roots systems is assumed to be a hemisphere shape as shown in Fig.3. Namely, it is assumed that areas and average diameters of roots have constant values at the location where the distance (r) from the point where the center line of the stump and ground surface line intersect is the same.

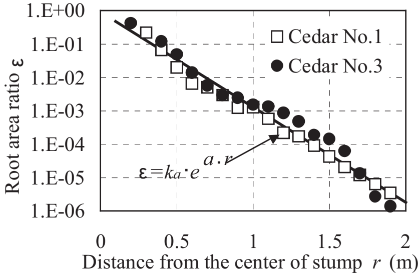


Fig.4: Relationship between root area ratio and radius r

Using two Japanese cedars aged 35 (Cedar No.1 [DHB:15.5cm], Cedar No.3 [DHB: 15.0cm]) removed from the Experimental Forest, Mie Prefectural Science and Technology Promotion Center, area ratio (ε)of roots defined by the diameter and the number of roots can be determined as Equation(6), as follows:

$$\varepsilon = Ar/As \tag{6}$$

where Ar is amount area of roots obtained from area of roots on the surface (its area is As) of hemisphere with the distance r (i.e. radius) from the center of the stump.

Fig.4 shows the relationship between the area ratios (ε) of roots and the radii r from the center of the stumps. The vertical axis indicates the area ratio (ε) expressed as a scale of logarithms. This figure illustrates that the area ratio (ε) of roots can be expressed by the approximate equation written as

$$\varepsilon = ka \cdot e^{a \cdot r} \tag{7}$$

where ka=0.907 , a=-6.57 expressed by the dimension unit of m.

There should be a great number of roots in the area (As) of hemisphere being distance (radius) r from the center of the stump. Though the diameters of roots, of course, are different each other, to the advantage of calculations, all the diameters are assumed to be represented by an average (dm). The average diameter is converted by using both amounts of area (Ar) and the number (N) of roots in the hemisphere surface at distance r. Then, the dm can be written as:

$$dm = \{4 \cdot Ar/(\pi \cdot N)\}^{1/2} \tag{8}$$

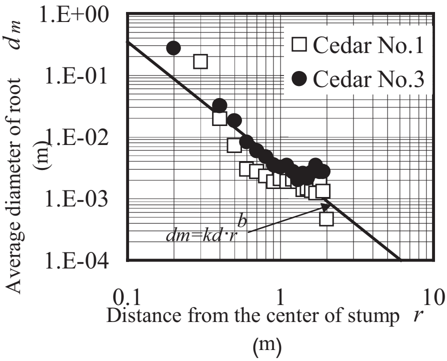


Fig.5: Relationship between average diameter of root and radius r

As shown in Fig.5, the variation of the average diameter (dm) depending on the r determined by actual datum, as mentioned above, may be expressed by the function of exponent as written by

$$dm = kd \cdot r^{b} \tag{9}$$

where $kd = 3.624 \cdot 10^{-3}$, $b = -1.984$ expressed by the dimension unit of m. The number (n) of roots per unit area can also be given by

$$n = N / As \tag{10}$$

Using Equation(8) and Equation(10), n can be written as following equation.

$$n = 4 \cdot \varepsilon \ /(dm^{2} \ \cdot \pi \ ) \tag{11}$$

Therefore using Equation(3) and Equation(11), the pull-out strength (st) per unit area of the tree roots system in the surface at distance r can be given by

$$st = n \cdot tc = 4 \cdot kq \cdot \varepsilon \ \cdot dm^{c-2} / \pi = 782 \cdot \varepsilon \ \cdot dm^{-0.55} \tag{12}$$

By using Equation(12), the reinforcement due to the tensile strength of the tree roots system (i.e. the root shear resistance stress (cr)) can be written by

$$cr = k \cdot n \cdot tc = k \cdot s_{t} = 782 \cdot k \cdot \varepsilon \ \cdot dm^{-0.55} \tag{13}$$

Equation(13) can provide the root shear resistance stress (cr) per unit area on the point of the distance r from the center of the stump by substitution of the area ratio (ε) in Equation(7) and the average diameter (dm) in Equation(9). The shear strength (τ) of the soil including the effect of the tree roots system can also be given by substitution of Equation(13) into Equation(5).

INCORPORATION OF ROOT RESISTANCE INTO STABILITY ANALYSIS

Equation (13) derived in the previous chapter prescribing the shear strength of the soil, including the effect of the tree roots system, will be incorporated into the Spencer method for two dimensional analysis of the slope stability.

It was assumed that the area ratio (ε) and the average diameter (dm) of the tree roots system vary as a hemisphere shape within the soil mass. Therefore, in principle, the variations in area ratio and average diameter with respect to three dimensional depth of the cross section of the slope, cannot be taken into account in two dimensional analysis. In this paper, since the major objective is to disclose the role of the tree roots system in slope stability, variations with respect to three dimensional depth of the cross section is not taken into account. Three dimensional treatment will be the subject in the future.

In general, the two dimensional methods for slope stability analysis may underestimate the effect of shear resistance in the side of slope failure shape; on the other hand, with respect to the tree roots system, two dimensional treatment may overestimate. Therefore, notice that results obtained from this study should be interpreted as the role of the tree roots system.

If locations of trees on the slope and slip surface are given, the root shear resistance stress (cr) can be obtained from Equation(13) using the r being distance from point at the slip surface to the point of the center of stump by substitution of the area ratio (ε) in Equation(7) and the average diameter (dm) in Equation(9) as mentioned above. The Spencer method being one of the slice methods, which divides a slope into a series of slices using vertical planes, assumes the base of each slice to be a straight line. As the width of the base of slice is finite, the radii r from the center of stump must vary depending on the position of points at the base of a slice. Therefore the value of the root shear resistance stress (cr) on the base of slice is obtained from integrating along the base plane numerically.

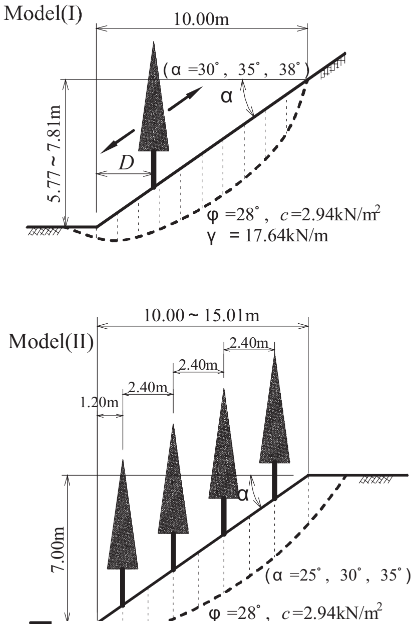


Fig.6: Section models
(Model(I): upper, Model(II): lower)

OPTIMIZATION OF SAFETY FACTOR

Of course, slope failure occurs with generating the slip surface. The slip surface formed by the shape and the location should be defined as one called the critical slip surface, indicating the lowest safety factor for the slope under the given conditions of the geographical features and the soil strength parameters, etc. For a slope with trees, the slip surface must be determined under the action of the root shear resistance force in addition to the above conditions. The method to determine the critical slip surface (i.e. the method to optimize the safety factor) adopts the Quasi Newtonian method (Kondo *et al.*, 1997). In this method, a combination of points shaping the slip surface indicating the lowest safety factor are determined by varying these points. In the analysis, both sides of the end points of the slope surface move along the ground surface, and the points on the vertical planes of the slice move up and down along the planes. The root shear resistance stress (cr) obtained from Equation(13) is determined in the correspondence with the location of the slip surface assumed in the analysis.

SECTION MODELS TO APPLY

The variation of the stability and the shape of the slip surface caused by the presence of root shear resistance force, will be described by analyzing two model sections consisting of homogeneous and isotropic soil material.

Section model(I)(hereafter called model(I)) is composed of three slopes inclined at 30deg., 35deg. and 38deg. with a horizontal plane in front of the toe of the slope, as shown in Fig.6.

Using model(I), an examination will be conducted with respect to changing the location of a tree on the ground surface of the slope. In the analysis, part of the slope is divided into 10 slices.

Section model(II) (hereafter called model(II)) is composed of three slopes 7m in height inclined at 25deg., 30deg. and 35deg. as shown in Fig.6. Using model(II), an examination will be conducted with respect to whether there is a group of trees or not. The horizontal interval of trees in the group is 2.4m, which is a standard value for the Japanese cedar aged 35, and the lowest tree is located at a 1.2m horizontal distance from the toe of the slope. The number of trees in the model(II) in the slope inclined at 25deg. is six; in the slope inclined at 30deg., five; in the slope inclined at 35deg., four. In the analysis, the weight of trees is considered as 0.49kN/m^2, and part of the slope is divided into 8 slices. In both model(I) and model(II), the unit weight of soil (γ) is 17.64kN/m^3, the internal friction angle (φ) is 28deg. and cohesion (c) is 2.94kN/m^2.

In the analysis of optimization, in case of model(I), the tip in the top of the slip surface will be fixed at a point of a 10m horizontal distance from the toe of slope, and the tip of the lowest slip surface will move along the line of the horizontal ground surface in front of the toe of slope (see Fig.6). In case of model(II), the tips in both the top and the lowest point of the slip surface will move along the line of the ground surface.

The k in Equation(1), which is necessitated in deriving the root shear resistance force on the slip surface, is 1.12 that is a average value computed by varying in the range of the θ_r from 48deg. to 72deg. when the φ is 28deg, based on the results by Wu *et al.*(1979).

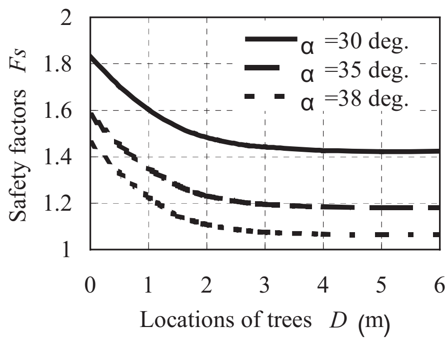


Fig.7: Variation of safety factors depending on distance D (Model(I))

COMPUTATION RESULTS AND DISCUSSION

Computation results and discussion will show below using the model (I) and (II).

Fig.7 concerning model(I) indicates the variation of values of safety factors depending on the locations of trees. In this figure, as the same soil strength parameters were utilized in all in the sections, of course, the safety factors become higher when decreasing the inclination angles of the slope. Against the variation of the distance D in the

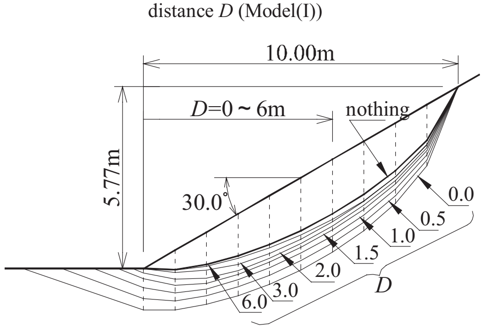


Fig.8: Variation of slip surface shape depending on distance D (Model(I)α =35deg.)

range from 0 to 6m, in the case of α =30deg., the safety factors vary in the range from 1.42 to 1.83; in the case of α =35deg., those vary in the range from 1.18 to 1.58; and in the case of α =38deg., those vary in the range from 1.06 to 1.46. The safety factors become higher with decreasing the distance *D*. And the variation rate of safety factors increases when decreasing the *D*.

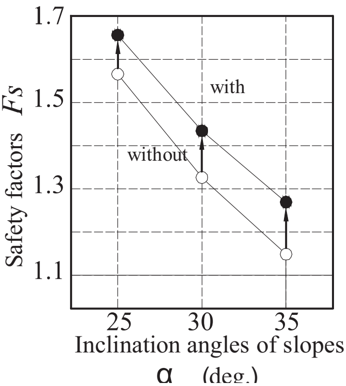


Fig.9: Difference of safety factors with and without a group of trees (Model(II))

Fig.8, as an example of the case of α =35deg., indicates the variation of the shapes of the slip surfaces depending on *D*. In this figure, accompanied with decreasing *D*, the location of lowest tip of slip surface making an appearance on the horizontal ground surface becomes far away from the slope, and the location of the slip surface become deeper as a whole. The rates of these variations increases with decreasing *D*, the same tendency as the case of the safety factor, as shown in Fig.7.

As shown in Fig.4, the value of root shear resistance stress (*cr*) increases rapidly with increasing the area ratio (ε) of a tree, accompanied with coming near the center of a tree stump. In the process of optimizing safety factor, the slip surface needs to be selected as deeper slip surface, because safety factor increases accompanied with slip surface being near the center of the tree stump. In case where a tree is located near the toe of slope (i.e. *D* is small), the minimum safety factor may increase by causing the passive failure phenomenon on the soil in front of the toe of the slope by the action of the slip surface being deeper.

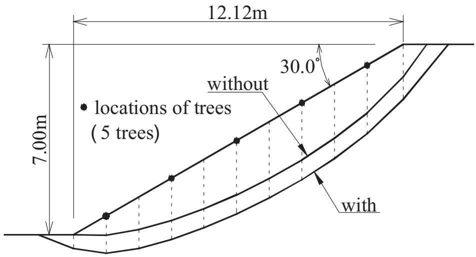


Fig.10: Difference of slip surface shape with and without a group of trees (Model(II)α =30deg.)

Fig.9 concerning model(II) indicates differences of the safety factors with and without a group of trees in the slope. In this figure, the white circles indicate the safety factors without a group of trees, and the black circles indicate one with a group of trees. As the shear resistance parameters are the same values in all the sections, the safety factors, as same as model(I), become higher when decreasing the inclination angles of the slopes.

In the case of α =25deg., the safety factors increase from 1.57 to 1.66 due to the group of trees; in the case of α =30deg., 1.33 to 1.44; in the case of α =35deg., 1.15 to 1.27, respectively. This figure shows that the increase rates of safety factors increase accompanied with increasing the inclination angle (α) of the slope, notwithstanding that the number of the trees is lesser(25deg.:six trees, 30deg.:five trees, 35deg.:four trees).

Fig.10, as an example of the case of α =30deg., indicates the difference in the shape of the slip surfaces with and without a group of trees. Similarly in model(I), the presence of the group of trees generates a deeper slip surface as a whole.

As the results of analysis using model (I) and (II), the role of the tree roots system in a slope is to increase the safety factor compared with a slope without; however, it also generates a deeper slip surface in which the value of safety factor is minimum, in other words, makes a scale of slope failure large if occurs. In a slope with a horizontal plane in front of the slope, the increase rate of safety factor increases accompanied with nearing the toe of the slope.

CONCLUSION

The role of the tree roots system was revealed in slopes consisting of homogeneous and isotropic soil material. To clarify the role, the formula for the root shear resistance stress was derived from the actual data of Japanese cedars; and the method, which incorporated the above formula into the Spencer method, was developed. By the numerical examination for two section models, the developed method revealed that the tree roots system can reduce the risk of slope failure; however, it induces an increase in the depth of the slip surface, namely makes a scale of slope failure large if occurs, that is fundamental role of the tree roots system. Summary in this study is as below:

1) The area ratio (ε) and the average diameter (dm) of roots in a 35-year-old Japanese cedar were qualified as the formula. These values are the function of the radius r of the hemisphere centered at the center of a tree stump.
2) The formula for the root shear resistance stress (cr) was derived from the values of the root ratio (ε) and the average diameter(dm) of roots.
3) The slope stability method which is able to optimize the safety factor in a slope with trees, which incorporated the formula for cr into the Spencer method, was developed.
4) The tree roots system induces the increase of the stabilization of slope failure; however, it also increases of the depth of critical slip surface, making the scale of slope failure large if it does occur.
5) If a tree is located at a lower part of a slope with a horizontal plane in front of the slope, the safety factor increases remarkably compared with other positions.

This study dealt with 35-year-old Japanese cedars using a two dimensional analysis method; however, to evaluate the effect due to other conditions and to accurately evaluate the effect of the actual phenomena, considering the points of view of the tree age, species and thinning grade, three dimensional analysis is necessary in that it deals with the shape of the slip surface and the distribution of tree roots system is modeled more accurately.

This paper is aimed at the factor of root reinforcement of soil for slope failure. Besides this, the following should be considered binding force to soil, soil moisture modification, wind throwing of tree, root wedging into a rock mass, etc., which may positively or negatively affect the stability for slope failure. In future, elucidation of the role of the tree roots system will be desired to be performed by synthetical examination considering the above factors.

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