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A SCALING MODEL OF DEPTH-DURATION-FREQUENCY RELATIONSHIP FOR THE ANALYSIS OF EXTREME RAINFALLS IN AN ALPINE REGION

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ABSTRACT

The relationship between rainfall depth, duration and frequency, represented in a compact form by the depth-duration-frequency (DDF) curve, has been of considerable interest to practising engineers and hydrologists for over a century. Recent research sought to apply the scaling hypotheses to annual maximum series of rainfall depth for different rainfall duration. It is shown here that, based on the empirically observed scaling properties of rainfall and some general assumptions about the cumulative distribution function for the annual maximum of the rainfall depth, it is possible to derive a simple DDF relationship. This general framework provides a basis for the generation of maps that can be used to infer DDF curves at any point of a particular area. Data from a dense raingauge network in a mountainous region in north-eastern Italy (the Trentino Province) are used to clarify the methodology for the construction and regionalization of the DDF relationship.
Key-words: Rainfall process, extreme rainfalls, statistical modelling, Gumbel distribution, scaling processes

INTRODUCTION

Design storms, defined as the rainfall intensity pattern for various probability of occurrences or return periods, are commonly required for designing hydraulic structures or for evaluating the effectiveness of a natural or manmade drainage system. The objective of rainfall frequency analyses is to estimate the amount of rainfall falling at a given point for a specified duration and return period. The frequency of the rainfall is usually defined by reference to the annual maximum series, which comprises the largest values observed in each year. An alternative data format for rainfall frequency studies is that based on the *peak-over-threshold* concept, which consist of all large precipitation amounts above certain thresholds selected for different durations. Due to its simpler structure, the annual-maximum-series-based method is more popular in practice.
Design storms are generally based on models linking rainfall depth, duration and frequency (DDF models). Such models enables the estimation of rainfall frequency for any intermediate duration. A further reason for developing and using a DDF model is to reconcile rainfall estimates for different durations. If each duration is treated separately, it is possible that

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contradictions between rainfall estimates of different durations could occur, for example if the estimated 12-hour 100-year rainfall were smaller than the 6-hour 100-year rainfall at the same site. The DDF relationship for a the return time T takes often the form of a power law relationship

$$h(d)_T = a_T d^{n_T} \tag{1}$$

where $h(d)_T$ is the rainfall depth at the specified return time T and duration d, and a_T and n_T are parameters. Power laws of this form are commonly used in Italy (Moisello, 1976; Della Lucia et al., 1976; Burlando and Rosso, 1996; Ranzi et al., 1999) and elsewhere (Chow et al., 1988; Reed and Stewart, 1994; Koutsoyiannis et al., 1998; IH, 1999). Note that these formulae are often expressed in terms of rainfall intensity rather than rainfall depth over a certain duration. The latter has been chosen here because it is cumulated rainfall depth, rather than instantaneous intensity, that is actually measured and that is of importance for flood estimation.

The traditional methodology to construct DDF curves has three main steps. In the first step, raw data are examined to obtain annual maximum rainfall depth series for each time duration. Then, a statistical analysis is carried out for each duration to estimate the quantiles for different return periods. Lastly, the DDF curves are usually determined by fitting a specified parametric equation for each return period to the quantiles estimates, using regression techniques. A high number of parameters is involved in this procedure. Usually, at least two parameters need to be estimated for each time duration, and other two for each DDF curve.

In contrast to the above treatments, several models have been recently developed which attempt to statistically and simultaneously match various properties of the rainfall process at different levels of temporal aggregation (Burlando and Rosso, 1996; Menabde et al., 1999; Ranzi et al., 1999; Nguyen et al., 2002; Veneziano and Furcolo, 2002). These procedures were based on the “scale invariance” (or “scaling”) concept. In this study, the scale invariance implies that statistical properties of extreme rainfall processes for different time scales are related to each other by an operator involving only the scale ratio. Two important practical implications of the scaling models of DDF curves are: (i) from a higher aggregation level it is possible to infer the statistical properties of the process at the finer resolutions, that may not have been observed; (ii) these models require a much smaller number of parameters with respect to the traditional heuristic methods used to derive DDF curves. However, note that a DDF model should not be expected to improve the accuracy of estimating rainfall for the primary durations at which data are available. Although a model incorporates information from other durations, Buishand (1993) showed that the dependence between annual maxima over different durations is such that little or no improvement can be obtained.

In this paper we will show how it is possible to derive a simple DDF relationship in the form of (1) based on empirically observed scaling properties and some assumptions about the cumulative distribution for the annual maximum of rainfall depth. Furthermore, we will show how it is possible to use the above model to study the geographical variation of DDF curves and, more specifically, to estimate maps that can be used to infer DDF curves at any point of a particular area. Data from a dense raingauge network in a mountainous region in north-eastern Italy (the Trentino Province, Figure 1) are used to clarify the methodology for the construction and regionalization of the DDF relationship.

Fig1: Study area location

THE SCALING CONCEPT

It has been repeatedly observed that, over a certain range of durations, the distribution of the random variable representing the maximum rainfall depth of duration *D* observed in each year (*H(D)*)*,* satisfies the scaling relation

$$H(\lambda D) \stackrel{d}{=} \lambda^{n} H(D) \tag{2}$$

where $\stackrel{d}{=}$ denotes equality in the probability distribution, λ denotes a scale factor and *n* is a scaling exponent, typically ranging from 0.3 to 0.5 (Burlando and Rosso, 1996; Menabde et al., 1999). This also implies that the raw moments of any order are scale-invariant, that is

$$E\left[H(\lambda D)^{q}\right] = \lambda^{qn} E\left[H(D)^{q}\right] \qquad q = 1,2,3... \tag{3}$$

where *q* denotes the order of the moment and *E[ ]* is the expectation operator. This property is defined as "wide sense simple scaling" (Gupta and Waymire, 1990).
The only functional form of $E[H(D)^q]$ satisfying (3) is

$$E\left[H(D)^{q}\right] = f(q) D^{-qn} \tag{4}$$

where *f(q)* is some function of *q*. Notice that if the exponent in (4) is not a linear function of *q*, in such cases the process is said to be "multiscaling" (Gupta and Waymire, 1990).

In order to exemplify the scaling property, we have analysed a set of annual maxima from the Trento precipitation station over eight durations (from 15 minutes to 24 hours), with lengths varying from 22 years (at 45 min) to 57 years (24 hours). Figure 2 represents the scaling behaviour of the moments, defined by (4). It can be seen that this data set shows very good scaling behaviour in the range of durations considered. The scaling exponents were calculated by regression of the raw moments of order q against duration (after logarithmic transformation) and are shown in Figure 3. Their dependence on q is precisely linear, thereby confirming the hypothesis about wide sense simple scaling. For the Trento station, the scaling exponent n is equal to 0.399.

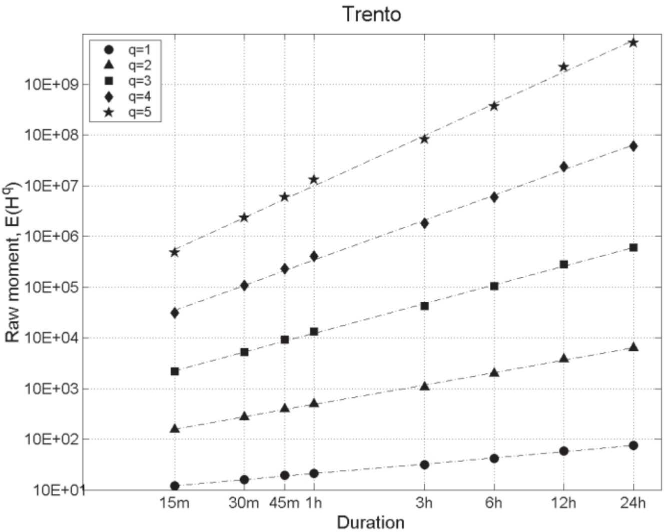


Fig2: Scaling of the moments with durations for Trento.

A GUMBEL SCALING MODEL OF DDF RELATIONSHIP

We assume here that wide sense simple scaling holds for a certain considered range of durations. Based on the scaling property defined above, and taking $\lambda=d/D$, one obtains for the first two moments

$$
\begin{aligned}
E[H(d)] &= d^n E[H(D)] \\
Var[H(d)] &= d^{2n} Var[H(D)] \\
CV &= \sqrt{\frac{Var[H(d)]}{E^2[H(d)]}} = cost
\end{aligned}
\qquad (5)
$$

Based on (5), the coefficient of variation, as well as the coefficient of skewness and the coefficient of kurtosis, is independent of duration d.

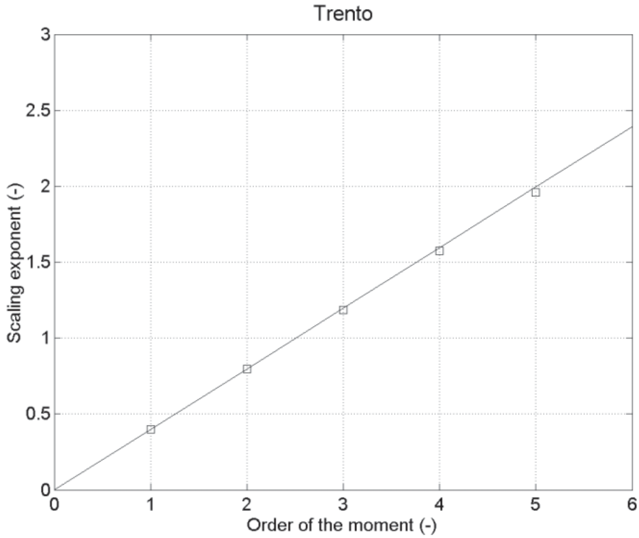


Fig3: Scaling exponents for Trento.

If we denote with a_1 the mean of the annual maximum rainfall depth for the specified temporal duration assumed as reference duration D (e.g., the hourly duration, if d is measured in hours), equation (5) can be written as

$$\begin{aligned} &E[H(d)] = a_1 d^n \\ &CV = cost \end{aligned} \qquad (6)$$

The values of a_1 and n can be estimated by linear regression of expectations of rainfall depth against duration, after log transformation, whereas the value of CV can be obtained as the average of coefficient of variation computed for the different durations, in the range of durations for which the scaling property holds.

The Gumbel or extreme value type 1 distribution has been used extensively in hydrology to model flood flows and extreme rainfall depths (Chow et al., 1988; Stedinger et al., 1993). The Gumbel cumulative distribution function can be easily inverted to provide the *Tth* quantile of Gumbel random variable for duration d in the following form

$$\begin{aligned} &h_T(d) = \mu(d)\left[1 - \frac{CV(d)\sqrt{6}}{\pi}[\varepsilon + y_T]\right] \\ &where \\ &y_T = ln\left(ln\left(\frac{T}{T-1}\right)\right) \end{aligned} \qquad (7)$$

where $\mu(d)$ is the mean and ε is Eulers constant, approximately 0.5772157. The relationships (6) can be substituted in (7) to obtain

$$h_T(d) = a_1\left[1 - \frac{CV\sqrt{6}}{\pi}\left[\varepsilon + y_T\right]\right]d^n \tag{8}$$

which represents the Gumbel simple scaling model of DDF curves.
L-moments estimators for the Gumbel distribution have been found better than product-moment estimators when observations are drawn from a range of reasonable distributions for hydrological variables (Stedinger et al., 1993). The Gumbel simple scaling model of DDF curves can be written as follows:

$$h_T(d) = a_1\left[1 - \frac{\tau_2}{ln(2)}\left[\varepsilon + y_T\right]\right]d^n \tag{9}$$

where τ_2 denotes the *L*-moment coefficient of variation (Stedinger et al., 1993), obtained as the average of *L*-moment coefficient of variations of the various durations.
The result of the application of the Gumbel simple scaling model to Trento rainfall depth data is shown in Figure 4, where the model estimated probability are compared with the sampling cumulated frequency of observed data. The Gumbel model was applied in this case with the product moment estimation procedure.

SPATIAL ESTIMATION OF DDF CURVES

The general framework described above provides a sound basis for evaluating the spatial variability of DDF curves in a given region. The idea is to study the variation of the DDF model parameters (a_1, n and CV), instead of the variation of the rainfall quantiles (as is usually done with the traditional approach).
The study area for this demonstration is the Trentino Province, with an area of around 6000 km^2. The area is characterized by a mountainous terrain, with altitudes ranging from 200 to 3600 m a.s.l.. The area lies at the southern border of the “inner alpine province” with its own distinctive climatic characteristics (Fliri, 1975). The inner alpine province is characterized by low precipitation amounts, due to the dual sheltering effect of the ranges to both the north and south. The south-eastern part of the Trentino Province is characterized by higher precipitation amounts, and commonly experiences showery precipitation, with thunderstorm and hail, particularly in summer and autumn. Thus the annual rainfall varies from about 1300 mm in the south-eastern part of the area to about 900 mm in the northern part.

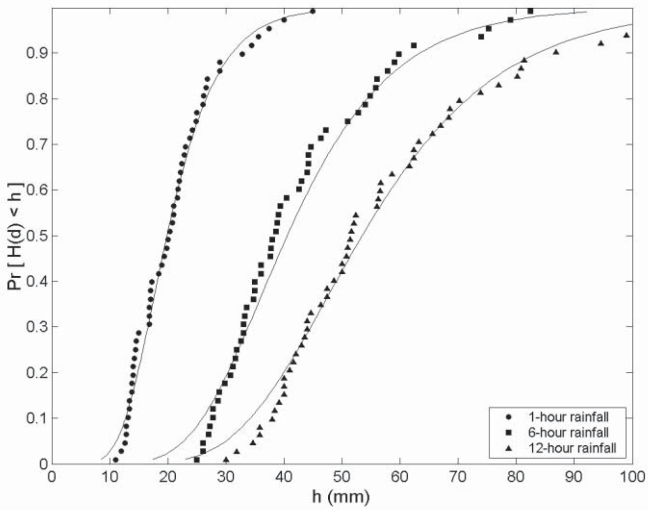


Fig4: Cumulative distribution frequency for the annual maximum rainfall depth for Trento station, by using the simple scaling Gumbel distribution.

Records of annual maximum rainfall depth at 91 recording stations were used. The durations examined were 1, 3, 6, 12 and 24 hours. In the first stage, we fitted the Gumbel simple scaling DDF model to the data. This model was considered suitable, since the Gumbel distribution was found (using Kolmogorov and Filliben test) to be appropriate for almost all the stations. Further than Kolmogorov and Filliben tests, we evaluated the appropriateness of using the Gumbel distribution by using a test on the GEV (Generalized Extreme Value) shape parameter *k*. As shown by Hosking et al. (1985) when data are drawn from a Gumbel distribution, the resultant *L*-moment estimator of *k* has mean 0 and variance

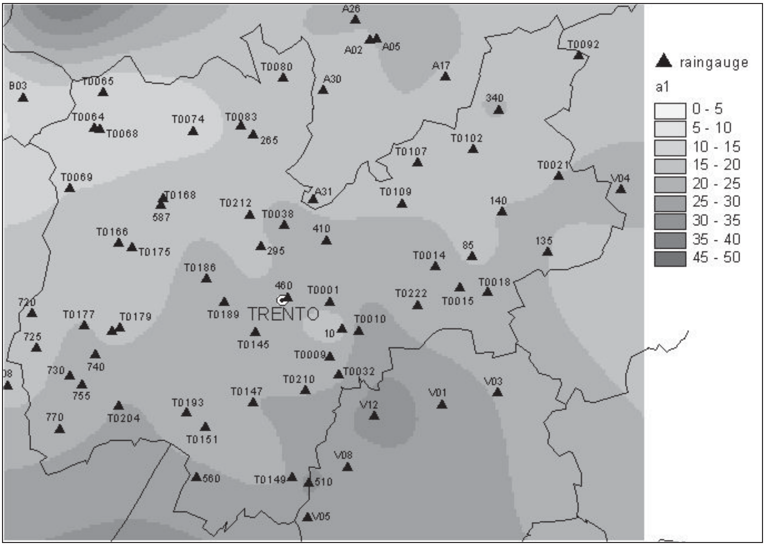
$$Var(k) = \frac{0.5633}{n} \qquad (10)$$

where *n* is the length of the sample. This allows the construction of a powerful test whether *k*=0 (i.e, appropriateness of the Gumbel distribution; null hypothesis) or not (alternative hypothesis). Applied to the data from the study region and for the various durations, this test results to no rejection of the null hypothesis.

Tests were carried out to check the validity of the simple scaling assumption (Ranzi et al., 1999), which was not rejected on most of the stations. Two parameters of the DDF model (a_1 and n) were estimated based on stations with at least 15 observations for each duration (55 stations), whereas the third model parameter (*CV*) was estimated based on stations with at least 25 years for each duration (43 stations).

A kriging procedure (Borga and Vizzaccaro, 1997) was used to estimate the spatial distribution of the three parameters across the area (Figures 5 and 6). These estimates can be used to generate DDF curves at any point in the region.

a)



b)

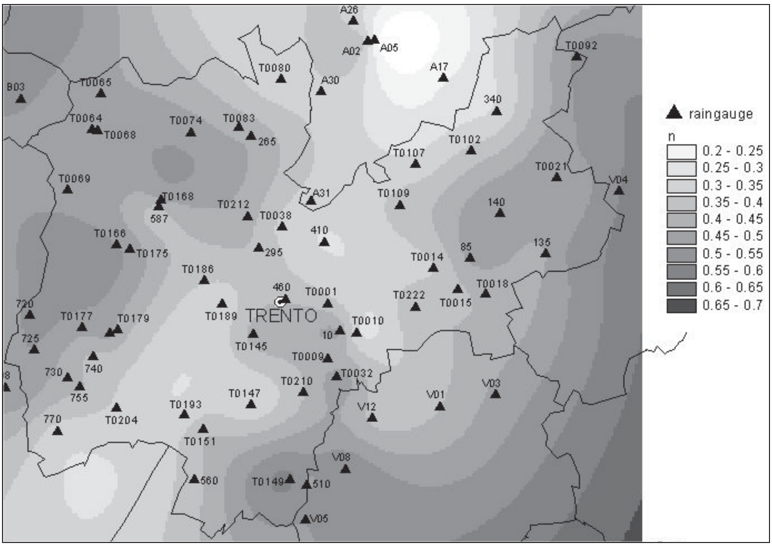


Fig5: Spatial variation of parameters a) a_1 and b) n

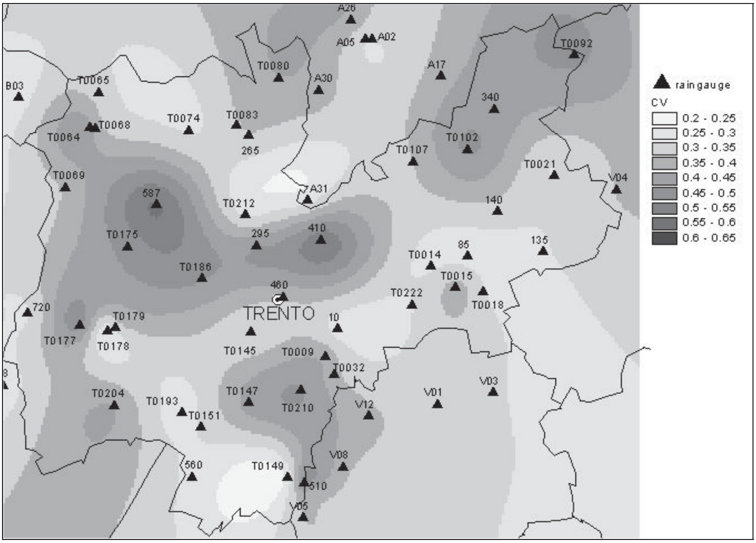


Fig6: Spatial variation of parameter *CV*

The overall performance of the procedure used to estimate DDF curves across the region was assessed by counting the exceedances of certain quantiles and comparing observed and expected exceedances (Buishand, 1991). DDF curves were estimated at each station of the network (without using the local data); then, the number of exceedances of rainfall depths with various rarities was counted. This test was carried out for T = 50, 100, and 200 years. The expected number of exceedances is obtained by dividing the total number of station years by T. This is based on the fact that for the annual maximum $H(d)$ at a certain site

$$Pr[\,H(d) > h_T\,] = \frac{1}{T} \tag{11}$$

For example, for T=100 years, the observed number of exceedances (over all the durations) is equal to 113, while the expected number is equal to 126 (since the total station years are 12579). This result shows that there is a small bias, with 11% fewer exceedances observed than expected according to the method. However, the difference is only slightly more than the binomial standard deviation (equal to 11.2), given by $[\,M(1-1/T)\,]^{0.5}$, where M is the expected number of exceedances.

It should be noted that this methodology allows to use daily rainfall maxima in the process of DDF estimation, provided that adjustments are made to account for the fact that they are fixed-interval rainfall amounts. The incorporation of data from the more dense network of non-recording stations may provide more detailed information on the geographical variation of DDF curves, which is a valuable attempt, particularly in a mountainous region. A more detailed analysis of this last point is reported in Borga et al. (2003).

CONCLUSIONS

The traditional framework of DDF estimation is not free of empirical considerations, which may be inconsistent with the probabilistic foundation of DDF relationships. The use of the scale invariance concept on the DDF derivation represents an attempt to cast the DDF relationship within a general mathematical and physical framework. The simple scaling framework also yields a more parsimonious model parametrisation for DDF relationship with respect to the traditional one, based on regression of quantile estimates against duration.
The simple scaling hypothesis has been found to describe reasonably well the DDF characteristics of rainfall depth annual maxima from a network of raingauges in an alpine region. The assumption of wide sense simple scaling lead to a simple form of the DDF relationship, based on the Gumbel distribution. The efficient parametrisation of the model (based on only three parameters) allowed the study of the geographical variation of DDF curves, based on the analysis of the parameters of the DDF curves, instead of the variation of rainfall depths. The results obtained show that the scaling framework offers a good basis for the regionalisation of DDF curves. Furthermore, this methodology allows the incorporation of data from the more dense network of non-recording stations in the process of DDF relationship construction. This results in a more detailed information on the geographical variation of DDF curves.

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